

Multi-Point Geometric Optimization of NACA Airfoils Across Multiple Reynolds Numbers and Angles of Attack

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Abstract

Single-point airfoil optimization targets one operating condition, which limits how reliably a design performs across real flight conditions. This study presents a multi-point optimization of a NACA 4-digit airfoil, maximizing lift-to-drag ratio and lift coefficient while minimizing drag coefficient, by varying maximum thickness, camber and camber position across five Reynolds numbers (1×10^5 to 2×10^6) and angles of attack from 0° to 12° in 2° increments. A benchmark was first selected from NACA 0012, 2412 and 4412 using TOPSIS, which identified NACA 4412 as the baseline. Two rounds of optimization followed. Latin Hypercube Sampling generated candidate geometries, which were ranked with TOPSIS, and the rankings were tested for robustness against weighting assumptions using Monte Carlo and one-at-a-time sensitivity analyses [1]. The round 1 optimum, `airfoil_29` (maximum thickness 11.28%, maximum camber 4.57%, camber position 37.21%), achieved the highest TOPSIS score (0.6143) in a final combined evaluation against the round 2 optimum, `refine_13` (0.6049), and NACA 4412 (0.5989), and was the most robust under sensitivity analysis. Relative to NACA 4412, `airfoil_29` performed better at Reynolds numbers of 3×10^5 and 1×10^6 , while NACA 4412 performed better at 1×10^5 , 5×10^5 and 2×10^6 .

Keywords

Airfoil optimization; TOPSIS; Latin Hypercube Sampling; Design of Experiments; Reynolds number; XFOIL; Sensitivity analysis; Multi-point optimization; angle of attack

1 Introduction

An airfoil is the two-dimensional structural profile of aircraft wings, propeller blades, and turbine rotors. Engineered with distinct leading and trailing edges, its primary function is to maximize aerodynamic efficiency by achieving a high lift coefficient (C_L) while minimizing the corresponding drag coefficient (C_D). This efficiency matters directly: small improvements in C_L/C_D translate into reduced fuel consumption, lower production and material costs, and better overall aircraft performance over an aircraft's operating lifetime [2].

Airfoil optimization is the process of modifying an airfoil's geometric characteristics — most commonly camber and thickness distribution — to improve this lift-to-drag performance under specified operating conditions. The task is inherently difficult because airfoil shape and aerodynamic performance are tightly coupled: small changes in geometric design variables can produce disproportionately large changes in C_L and C_D . As a result, optimization must evaluate a wide range of design-variable combinations precisely before a candidate shape can be confidently identified as optimal. Given how directly aerodynamic efficiency affects fuel use, cost, and environmental impact, the development of rigorous airfoil optimization techniques remains an active area of aerospace engineering research.

In existing works, there is not a specific mention about optimization across low and high Reynolds numbers set as one range. Hence, this paper evaluates the optimized combinations for a standard NACA airfoil for better performance across low and high Reynolds numbers both. This ensures that there is significant performance for the airfoil across the Reynolds number range. This can reduce production costs as a different airfoil needn't be produced to operate at different Reynolds numbers and 1 airfoil is sufficient for a large range.

The aim of this paper is to produce an optimized airfoil based on a standard NACA airfoil, balancing aerodynamic performance with potential reductions in material and production costs. TOPSIS is used as the primary method for evaluating and ranking candidate airfoils, while OAT and Monte Carlo sensitivity analyses are used to assess the robustness of closely ranked solutions to changes in weighting. LHS and DOE are employed to systematically explore and obtain the design space across multiple optimization rounds.

Section 2 provides the literature and background review, presenting past work on the field of airfoil optimization and methods that were used to find the optimized airfoil. Section 3 justifies the methodological choices used in this study; Section 4 entails the methodology for this multi-point optimization; Section 5 presents the results and analysis of the results; Section 6 concludes the paper

2 Literature and background review

Optimization is a vast field in aerospace, and different types of solution algorithms have been used to produce optimized airfoils: MOPSO (multi-objective particle swarm optimization method) [3], by forcing a digital swarm of trial shapes to cooperate; surrogate-based optimization [4], where fast math models replace slow fluid simulations to design airfoils; and the adjoint approach in gradient-based optimization [5], which calculates sensitivities of all shape variables simultaneously using backward-step mathematics.

Separately from the choice of solution algorithm, an optimization problem can also be structured around how many operating conditions its objective function is evaluated against. This paper uses multi-point optimization [6], which evaluates a design's performance across multiple operating conditions simultaneously — such as several Reynolds numbers or angles of attack — rather than a single fixed condition, and can, in principle, be paired with any of the algorithms described above.

There have been multi-point optimizations on varying Reynolds numbers: low Re optimizations [6], high Re optimizations [7]. Prior work has been done on low and high Reynolds numbers but not on both at the same time

- XFRL5 - XFRL5 is a free wing and aircraft computational software operating at low Reynolds numbers. Design and analysis are based on Lifting Line Theory, Vortex Lattice Method, and 3D Panel Method which uses XFOIL as a 2-D model software [8]
- XFOIL - XFOIL is an interactive program for the design and analysis of subsonic airfoil shapes, developed by Mark Drela at MIT. It combines a high-order panel method with a robust boundary layer formulation to predict lift, drag, and pressure distributions accurately at low Reynolds numbers [9]

3 Justification of Methodological Choices

3.1 Benchmark Airfoil Selection

A multi-criteria decision-making method was required to select the benchmark airfoil, since no single aerodynamic metric fully captures overall airfoil suitability [10]. The three candidate airfoils were selected to be NACA 0012, 2412, and 4412, because their camber values (0%, 2%, and 4% respectively)

span a representative range of camber-driven aerodynamic behavior, providing a benchmark relevant to assessing performance across a range of operating conditions. XFLR5-based selection methodologies of this kind are established in low-Reynolds-number airfoil design research [8].

3.2 Design and Geometric Variables

Maximum camber, maximum thickness, and camber position were the three geometric variables varied to produce the optimized airfoil, as these directly govern the characteristics of lift and drag [11,12]. Thickness position was held constant at its NACA 4412 value (30% of chord) to isolate the effects of the three variables under investigation.

3.3 Aerodynamic Evaluation Tool

XFOIL [9] was selected as the aerodynamic solver due to its established use in low-Reynolds-number airfoil analysis, its significantly lower computational cost compared to other CFD methods, and its accessibility given the scope and time frame of this study.

3.4 Design of Experiments (Latin Hypercube Sampling)

A Design of Experiments (DOE) approach based on Latin Hypercube Sampling (LHS) [13] was used to generate candidate airfoil geometries for both optimization rounds – centered on NACA 4412 in the first round and on the round 1 optimum (airfoil_29) in the second. LHS was selected over simple random sampling because it ensures more uniform coverage of the design space for a given number of samples, reducing clustering or gaps in the sampled region.

3.5 Sample Sizes and Number of Airfoils

Sample sizes of 30 and 15 airfoils were used for the first and second round optimizations respectively. These sizes were selected to sample the design space at intervals fine enough to capture meaningful differences in aerodynamic performance between candidates, while avoiding an impractically large number of XFOIL evaluations given the computational cost of running each candidate across five Reynolds numbers and seven angles of attack, within the timeframe of this study.

3.6 Percentage Alterations

Design variable bounds of $\pm 20\%$ (first round) and $\pm 5\%$ (second round) relative to each round’s center point were used. The $\pm 20\%$ range was chosen to cover a meaningful region of the design space around NACA 4412, while remaining narrow enough that generated airfoils stayed within the aerodynamic character of the NACA 4-digit family rather than converging toward geometries resembling unrelated airfoil types. The second round was re-centered on airfoil_29, the round 1 optimum, and narrowed to $\pm 5\%$ to locally refine the design in the immediate neighborhood of the strongest-performing candidate identified in round 1.

3.7 Multi-Criteria Scoring (TOPSIS)

The Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) was selected to rank candidate airfoils because it allows multiple, potentially conflicting performance criteria to be combined into a single comparable score, based on each candidate’s geometric distance from an ideal and a worst-case solution [10].

3.8 Criteria Selection

Four criteria were used in the TOPSIS evaluation: mean lift coefficient, mean drag coefficient, mean lift-to-drag ratio, and the coefficient of variation (CV) of lift-to-drag ratio. The first three represent standard measures of aerodynamic efficiency. The CV of L/D was included specifically to capture performance consistency across the tested Reynolds numbers, directly reflecting this study’s main objective: assessing the robustness of performance across low and high Reynolds numbers.

3.9 Weights

Mean lift-to-drag ratio was assigned the highest weight (0.45) as the primary performance metric targeted by this study. Mean lift and drag coefficients (C_L and C_D respectively) were assigned weights of 0.25 and 0.20 as supporting performance measures. The CV of mean lift-to-drag ratio was assigned the lowest weight (0.10), reflecting its role as a criterion for assessing the robustness of already-produced design variable combinations, rather than a primary optimization objective.

3.10 Sensitivity Analysis (OAT and Monte Carlo)

Two complementary sensitivity analyses were conducted to test the robustness of the TOPSIS ranking to uncertainty in the criteria weights [14]. The Monte Carlo analysis tests robustness under simultaneous, randomized variation of all four weights, simulating realistic joint uncertainty. The one-at-a-time (OAT) analysis instead varies each weight individually while holding the others fixed, isolating the ranking’s sensitivity to each criterion independently.

3.11 Priority in Results

When the nominal TOPSIS ranking and sensitivity analysis disagree during the analysis of results, TOPSIS is given priority as the final measure of judgment, because it is the primary ranking method used by this study, while sensitivity analysis is used to check the robustness of the optimized airfoil(s) rather than serving as an alternate ranking method in its own right, since rankings under fixed weights can be sensitive to the specific weighting scheme chosen [15]. This priority rule is applied consistently whenever a tie-break is required in the Results and Discussion section.

4 Methodology

4.1 Selection of the Benchmark Airfoil from 3 standard NACA airfoils.

A benchmark airfoil was first required to serve as the baseline geometry for the optimization process. Three NACA airfoils (NACA 0012, NACA 2412, and NACA 4412) were selected as the benchmark airfoils as justified in section 3.1.

The airfoils were imported into XFLR5 by navigating to *Design* $\rightarrow$ *NACA Airfoils* and entering the corresponding four-digit airfoil designation. A batch analysis was then performed using Reynolds numbers of 1×10^5 , 3×10^5 , 5×10^5 , 1×10^6 , and 2×10^6 , while maintaining an angle of attack sweep from 0° to 12° with increments of 2° . The resulting polar data for each airfoil, created using the graphs in figure 1, 2, 3 were exported and subsequently processed to obtain the following performance metrics: mean lift coefficient (C_L), mean drag coefficient (C_D), mean lift-to-drag ratio (L/D), and the coefficient of variation (CV) of the lift-to-drag ratio, calculated at each Reynolds number across the 7 angles of attack and then averaged over the 5 Reynolds numbers. For each performance metric, the reported mean represents the unweighted arithmetic average calculated across all 35 operating conditions (5 Reynolds numbers $\times$ 7 angles of attack).

To enable a balanced multi-criteria comparison, the performance metrics were normalized prior to the application of the Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS) [10].

Normalization prevents criteria with larger numerical magnitudes, such as the mean lift-to-drag ratio, from dominating the distance calculations.

The normalization denominator for each performance metric was calculated using

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^m x_{kj}^2}} \quad (1)$$

where x_{ij} is the performance value of airfoil i for criterion j , m is the number of candidate airfoils, and r_{ij} is the resulting normalized value

where the denominator was obtained by combining the values of the three candidate airfoils for that particular metric. Each performance metric value was subsequently divided by its corresponding normalization denominator to obtain the normalized decision matrix.

TOPSIS Criteria Weights

Criterion	Weight
Mean CL (benefit)	0.25
Mean CD (cost)	0.20
Mean L/D (benefit)	0.45
Mean CV (cost)	0.10

Table 1: TOPSIS criteria weights used in the decision matrix.

The normalized values were then multiplied by the criteria weights in Table 1 to form the weighted normalized decision matrix using the equation:

$$v_{ij} = w_j r_{ij} \quad (2)$$

where w_j is the weight assigned to criterion j .

The positive ideal solution and negative ideal solution were subsequently determined by selecting the most favourable and least favourable values, respectively, for each performance metric across the three candidate airfoils.

The separation distances from the positive ideal (S^+) and negative ideal (S^-) were then calculated using equation for the distance to positive ideal:

$$S_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2} \quad (3)$$

where v_j^+ is the positive ideal value for criterion j and n is the number of criteria and equation for distance to negative ideal:

$$S_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2} \quad (4)$$

where v_j^- is the negative ideal value for criterion j

. Finally, the relative closeness to the ideal solution was determined using the equation:

$$C_i = \frac{S_i^-}{S_i^+ + S_i^-} \quad (5)$$

where C_i is the TOPSIS score for airfoil i , ranging from 0 (closest to the negative ideal) to 1 (closest to the positive ideal).

producing the TOPSIS score for each airfoil.

4.2 First-Round Airfoil Optimization

Following the selection of the NACA 4412 as the benchmark airfoil, a Design of Experiments (DOE) approach based on Latin Hypercube Sampling (LHS) [13] was used to generate the initial design population. Thirty airfoil geometries were selected for the first round of optimization to provide a representative sampling of the design space while maintaining a computationally feasible number of aerodynamic analyses.

The three geometric design variables, namely maximum thickness, maximum camber and camber position for the NACA 4 digit parametrization [11] were each permitted to vary by $\pm 20\%$ relative to the corresponding NACA 4412 values. The maximum thickness location was held constant at 30% chord, matching the configuration of the baseline NACA 4412 airfoil. The difference between the upper and lower bounds of each design variable was divided by thirty, then the division constant was added to the lower limit until it reached the upper limit to form equally sized intervals. A single random value was sampled from each interval to ensure uniform coverage of the design space. The sampled values for each design variable were subsequently randomized independently—preserving Latin Hypercube sampling characteristics—before being combined to generate thirty unique airfoil geometries.

The generated airfoil coordinates were analyzed using XFOIL under the same Reynolds numbers and angle of attack sweep employed during the benchmark airfoil selection. Owing to the large number of airfoils requiring analysis, the aerodynamic evaluation was automated using a custom Python script that generated NACA coordinate files for each candidate airfoil, executed XFOIL [9] via command-line calls in viscous mode, and parsed the resulting polar output files to extract C_L , C_D , and L/D at each Reynolds number and angle of attack. XFOIL was run with a panel count of 100, a transition criterion (N_{crit}) of 9, and a maximum of 200 iterations per operating point; cases that failed to converge within this limit were flagged and excluded from further analysis (see Section 6, Conclusion). To verify the Python/XFOIL pipeline against the manual XFLR5-based benchmark analysis, NACA 4412 was independently re-evaluated using a re-implementation of the automated script with identical settings, Reynolds numbers and angles of attack. The resulting C_L , C_D , and L/D values agreed with the XFLR5 batch analysis results to within 8.5% (mean difference 6.0%), with the pipeline giving consistently lower C_L and C_D . This offset is attributed to differences in solver implementation and paneling between the two tools.

The mean lift coefficient (C_L), mean drag coefficient (C_D), mean lift-to-drag ratio (L/D), and coefficient of variation of the lift-to-drag ratio were calculated for each airfoil. These performance metrics were subsequently evaluated using the same TOPSIS procedure described in Section 4.1, producing a relative TOPSIS score for each candidate airfoil.

Because the TOPSIS scores of the 29 evaluated (of the 30 generated) airfoils were relatively close to each other, two complementary sensitivity analyses were conducted to assess whether the TOPSIS-based

ranking was sensitive to the specific choice of criteria weights (Table 1): a Monte Carlo weight-perturbation analysis and a one-at-a-time (OAT) weight sweep [14]. This analysis assessed whether the highest-ranked airfoil remained the preferred solution under a range of plausible weighting combinations rather than relying on a single deterministic set of weights.

In the Monte Carlo analysis, each of the four criteria weights (Mean C_L , Mean C_D , Mean L/D , Mean C_V) was independently perturbed around its base value (Table 1) by a uniformly sampled amount within a fixed percentage of its base value, and the resulting four weights were renormalized to sum to 1. This process was repeated for 2,000 trials, and for each trial, the TOPSIS ranking was recomputed under the perturbed weight vector. The airfoil ranked highest in each trial was recorded, allowing the proportion of trials won and the average rank across all trials to be calculated for every candidate (Tables 4, 6, and 8).

In the OAT sweep, each of the four weights was varied individually across a fixed range in increments of 0.05, while the remaining three weights were held at their original base values without renormalization. The tested ranges were 0.05–0.40 for Mean C_L , 0.05–0.30 for Mean C_D , 0.30–0.55 for Mean L/D , and 0.05–0.20 for Mean C_V . At each step, the TOPSIS ranking was recomputed and the resulting top-ranked airfoil recorded (Table 9). Unlike the Monte Carlo analysis, the OAT sweep does not preserve the constraint that weights sum to 1; it is intended to isolate the individual sensitivity of the ranking to each criterion in turn, rather than to simulate a realistic joint perturbation of all weights.

4.3 Second round airfoil optimization and choosing final benchmark coordinates

airfoil_29, the highest-ranked design from the initial 30-airfoil DOE/TOPSIS optimization, was selected as the center point for a second, localized refinement stage. Using the same LHS/DOE methodology, 15 refined airfoils were generated within a narrower parameter range ($\pm 5\%$) surrounding airfoil_29's design point, and evaluated using the same XFOIL/TOPSIS pipeline with a Monte Carlo sensitivity.

To validate whether the two-stage optimization produced a genuine improvement, a final combined TOPSIS evaluation with sensitivity tests (OAT and Monte Carlo) was conducted incorporating all 15 refined airfoils alongside two reference points: NACA 4412 (an established baseline airfoil) and airfoil_29 (the first-stage optimum), enabling direct comparison against both an external benchmark and the method's own starting point.

5 Results and Discussion

The primary objective is to optimize the NACA 4412 airfoil to increase (L/D) ratio while also increasing lift coefficient (C_L) and minimizing drag coefficient (C_D). To determine the optimized airfoil, a two-stage method is used using the benchmarked 4412 NACA airfoil. Both stages use LHS DOE, TOPSIS and a Monte Carlo sensitivity test to determine the best performing airfoil out of them all. Since TOPSIS is a widely used method for ranking [10]

5.1 Benchmark airfoil selection

NACA Benchmark Airfoil TOPSIS Ranking

Rank	Airfoil	TOPSIS Score
1	NACA 4412	0.925
2	NACA 2412	0.539
3	NACA 0012	0.002

Table 2: TOPSIS scores of the 3 standard NACA airfoils

According to the TOPSIS results of the benchmark airfoil selection between NACA 4412, 2412 and 0012 from Table 2, NACA 4412 achieved the highest TOPSIS score with 0.925/1. NACA 4412 demonstrated a substantial performance margin with a TOPSIS relative closeness score of 0.925, compared to 0.539 for NACA 2412 and 0.002 for NACA 0012. Given this clear score separation, a sensitivity analysis to assess ranking robustness was not required for this baseline selection phase. This ordering is consistent with the published aerodynamic theory: lift generation increases as camber increases [12]; this is supported by the XFLR5 polar curve generation for the 3 NACA airfoils which show that NACA 4412 has the higher maximum peak at each of the angle of attacks in the experimental range for the C_L/a and $(L/D)/a$ curve compared to NACA 2412 and NACA 0012

For the symmetric NACA 0012 profile, the pipeline consistently reproduced the zero-lift condition ($C_L = 0$ at $\alpha = 0^\circ$) across the given range of Reynolds numbers, confirming that the automated script reproduces the expected symmetric-airfoil behaviour.

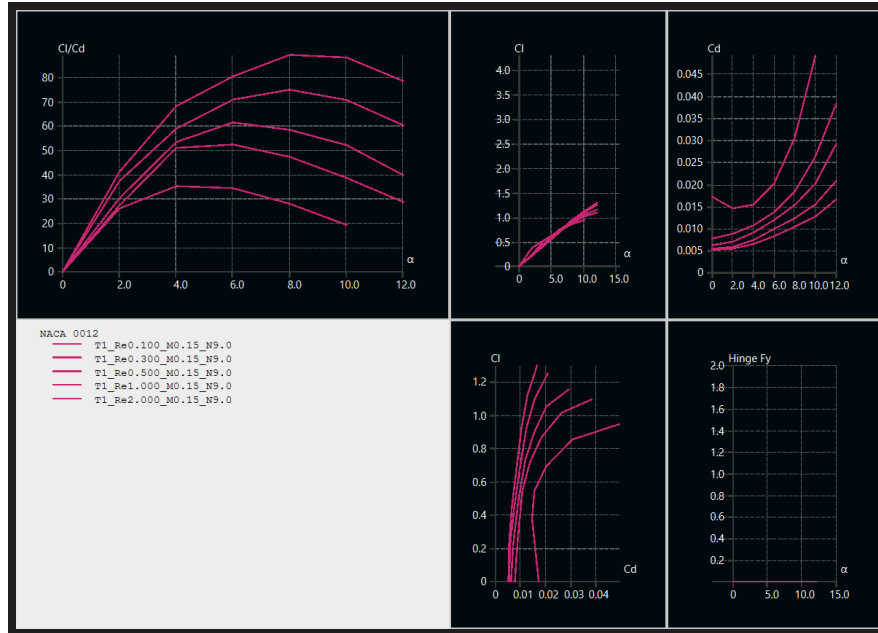


Figure 1: XFLR5 polar curves for NACA 0012 across the five tested Reynolds numbers.

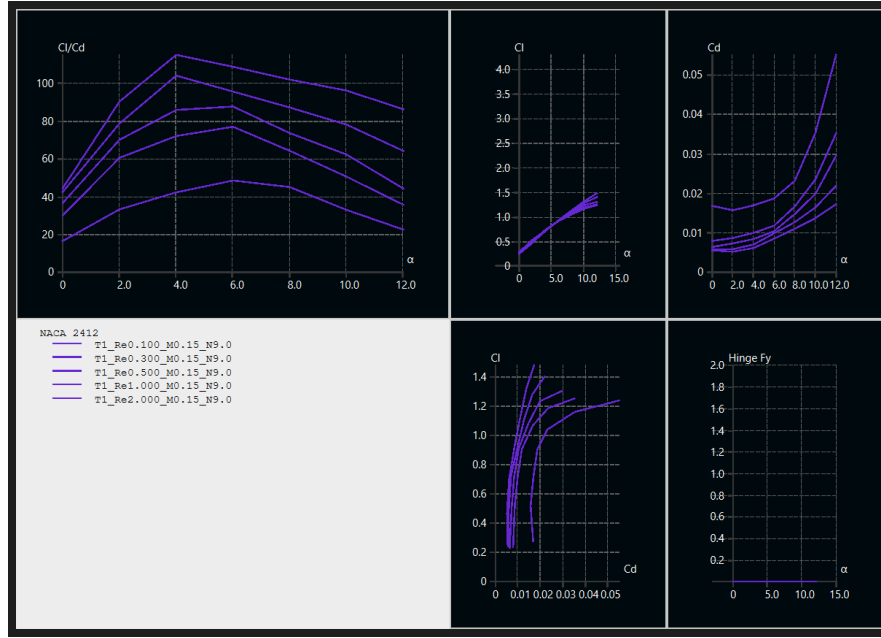


Figure 2: XFLR5 polar curves for NACA 2412 across the five tested Reynolds numbers.

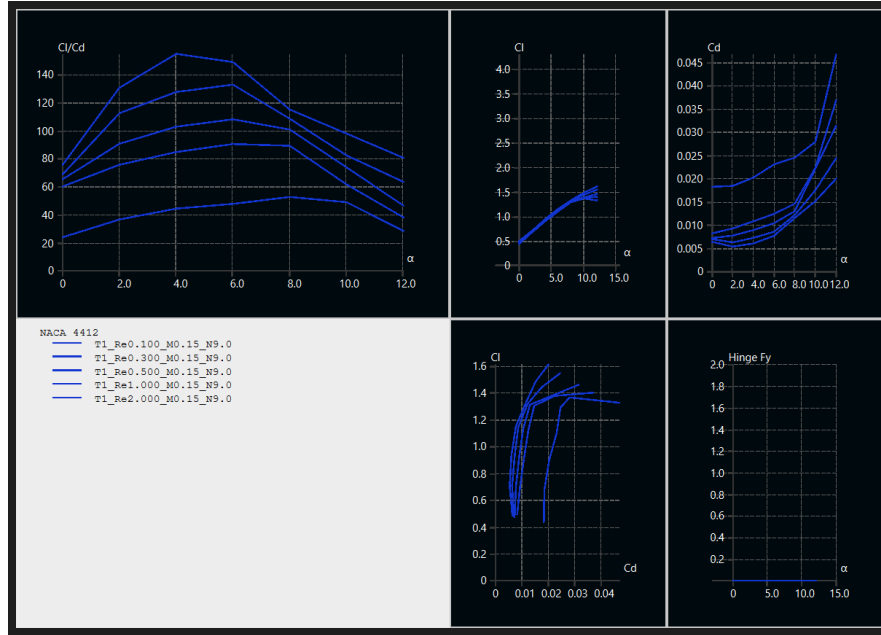


Figure 3: XFLR5 polar curves for NACA 4412 across the five tested Reynolds numbers.

5.2 First round optimization

Round 1: TOPSIS Ranking (30-Airfoil Global DOE)

Rank	Airfoil	S+	S-	TOPSIS Score
1	airfoil_29	0.005160	0.013436	0.722528
2	airfoil_06	0.004716	0.011564	0.710329
3	airfoil_21	0.005545	0.013478	0.708499
4	airfoil_30	0.004764	0.011465	0.706435
5	airfoil_23	0.005259	0.011379	0.683927
6	airfoil_28	0.005611	0.010529	0.652351
7	airfoil_26	0.006517	0.011715	0.642569
8	airfoil_08	0.005987	0.010198	0.630077
9	airfoil_20	0.006754	0.011470	0.629401
10	airfoil_22	0.006853	0.009330	0.576534
11	airfoil_04	0.007724	0.010090	0.566424
12	airfoil_24	0.007458	0.008881	0.543559
13	airfoil_17	0.009092	0.010172	0.528023
14	airfoil_14	0.007749	0.008656	0.527653
15	airfoil_03	0.008011	0.008263	0.507743
16	airfoil_16	0.008454	0.007547	0.471676
17	airfoil_07	0.008775	0.007600	0.464138
18	airfoil_12	0.009027	0.007466	0.452679
19	airfoil_05	0.008960	0.007288	0.448567
20	airfoil_27	0.011495	0.008563	0.426902
21	airfoil_09	0.010870	0.007834	0.418862
22	airfoil_19	0.010140	0.007259	0.417198
23	airfoil_02	0.009794	0.006977	0.416022
24	airfoil_13	0.009780	0.006584	0.402338
25	airfoil_01	0.010637	0.007119	0.400923
26	airfoil_25	0.010478	0.005747	0.354199
27	airfoil_15	0.012329	0.006738	0.353403
28	airfoil_10	0.013787	0.006015	0.303746
29	airfoil_11	0.011498	0.004833	0.295951

Table 3: Round 1 TOPSIS ranking of the 29 evaluated airfoils from the 30-airfoil global DOE

Round 1: Monte Carlo Sensitivity (2000 Trials)

Airfoil	Times Ranked #1	% of Trials	Avg Rank
airfoil_29	1334	66.70	1.72
airfoil_06	565	28.25	2.17
airfoil_30	70	3.50	3.23
airfoil_23	31	1.55	4.65
airfoil_01	0	0.00	24.00
airfoil_02	0	0.00	22.05
airfoil_03	0	0.00	14.28
airfoil_04	0	0.00	11.30
airfoil_05	0	0.00	19.19
airfoil_07	0	0.00	16.51
airfoil_08	0	0.00	8.10
airfoil_09	0	0.00	21.71
airfoil_10	0	0.00	28.30
airfoil_11	0	0.00	28.61
airfoil_12	0	0.00	18.32
airfoil_13	0	0.00	23.53
airfoil_14	0	0.00	12.76
airfoil_15	0	0.00	26.47
airfoil_16	0	0.00	16.65
airfoil_17	0	0.00	14.70
airfoil_19	0	0.00	21.56
airfoil_20	0	0.00	8.34
airfoil_21	0	0.00	3.35
airfoil_22	0	0.00	10.43
airfoil_24	0	0.00	12.64
airfoil_25	0	0.00	26.47
airfoil_26	0	0.00	7.18
airfoil_27	0	0.00	20.19
airfoil_28	0	0.00	6.62

Table 4: Round 1 Monte Carlo sensitivity results (2000 trials).

According to the TOPSIS results of the first round optimization selection in Table 3, airfoil_29 got the highest relative score to the best combination metrics. However, since the TOPSIS score of airfoil_29 was only 0.012199 higher than the 2nd airfoil, a Monte Carlo sensitivity test was needed to assess the robustness of the TOPSIS results.

In the Monte Carlo sensitivity test in Table 4, airfoil_29 won with an average rank 1.72, meaning that its closest to rank 1 (having the highest score during all permutations), (lower the better) which was sufficiently better than the 2nd airfoil which is airfoil_06 of average rank 2.17. Consequently, the Monte Carlo analysis reinforced the primary TOPSIS ranking, eliminating the need for rank reconciliation in this first round (unlike in Round 2, where discrepancies required further discussion).

5.3 Second round optimization

Round 2: TOPSIS Ranking (15-Airfoil Local Refinement)

Rank	Airfoil	S+	S-	TOPSIS Score
1	refine_13	0.002202	0.006139	0.736008
2	refine_07	0.002059	0.005704	0.734745
3	refine_09	0.002146	0.005152	0.705984
4	refine_01	0.003037	0.004686	0.606737
5	refine_04	0.003088	0.004752	0.606103
6	refine_12	0.003169	0.004873	0.605986
7	refine_03	0.002873	0.004375	0.603578
8	refine_11	0.003199	0.004637	0.591790
9	refine_10	0.003313	0.004715	0.587302
10	refine_05	0.003726	0.003949	0.514497
11	refine_15	0.004236	0.003663	0.463743
12	refine_14	0.004472	0.003841	0.461991
13	refine_06	0.004219	0.003595	0.460116
14	refine_02	0.005450	0.003045	0.358434
15	refine_08	0.006340	0.003259	0.339500

Table 5: Round 2 TOPSIS ranking of the 15-airfoil local refinement.

Round 2: Monte Carlo Sensitivity (2000 Trials)

Airfoil	Times Ranked #1	% of Trials	Avg Rank
refine_07	872	43.60	1.73
refine_13	817	40.85	1.75
refine_09	311	15.55	2.51
refine_01	0	0.00	6.09
refine_02	0	0.00	14.32
refine_03	0	0.00	6.11
refine_04	0	0.00	5.74
refine_05	0	0.00	9.94
refine_06	0	0.00	12.23
refine_08	0	0.00	14.58
refine_10	0	0.00	7.54
refine_11	0	0.00	7.14
refine_12	0	0.00	6.45
refine_14	0	0.00	12.04
refine_15	0	0.00	11.82

Table 6: Round 2 Monte Carlo sensitivity results (2000 trials).

A second round optimization is required as the difference between airfoil_29 and the next airfoil behind it in the rankings is 0.012199 (which is small), thus the combinations around airfoil_29 are to be optimized and tested to confirm whether a superior local configuration of airfoil_29 exists. The localized LHS was centered around a $\pm 5\%$ change as the initial $\pm 20\%$ change is too big for the current airfoil_29 and it also overlaps with some of the other airfoils tested in round one.

Looking at the TOPSIS ranking in Table 5, refine_13 achieved the highest TOPSIS score (0.736008), with refine_07 second (0.734745) — a difference of only 0.0013. This narrow margin required a sensitivity test. In the Monte Carlo sensitivity results (Table 6), refine_07 achieved a better average rank (1.73) than refine_13 (1.75), though these values are also very close. As established in the Justification of Methodological Choices section, TOPSIS is given priority as the primary decision criterion in such cases: refine_13 achieved a higher TOPSIS score than refine_07 under the study's nominal criteria weights, even though refine_07 was ranked highest in a larger proportion of the perturbed Monte Carlo trials. On this basis, refine_13 moves onto the final round of optimization.

5.4 Final validated comparison

Final Combined TOPSIS Ranking (15 Refine Airfoils + airfoil_29 + NACA4412)

Rank	Airfoil	S+	S-	TOPSIS Score
1	airfoil_29	0.004280	0.006817	0.614307
2	refine_13	0.004528	0.006933	0.604899
3	NACA_4412	0.005852	0.008740	0.598938
4	refine_07	0.004953	0.007043	0.587118
5	refine_09	0.004713	0.006340	0.573602
6	refine_12	0.005357	0.005688	0.515003
7	refine_03	0.005486	0.005726	0.510717
8	refine_04	0.005936	0.006155	0.509062
9	refine_10	0.006224	0.006425	0.507942
10	refine_11	0.006100	0.006216	0.504696
11	refine_01	0.005721	0.005808	0.503752
12	refine_05	0.006588	0.005716	0.464550
13	refine_14	0.007307	0.005806	0.442755
14	refine_15	0.007068	0.005529	0.438941
15	refine_06	0.006962	0.005148	0.425136
16	refine_02	0.007994	0.005206	0.394415
17	refine_08	0.007698	0.004103	0.347692

Table 7: Final combined TOPSIS ranking (15 refined airfoils, airfoil_29, and NACA 4412).

Final Combined Sensitivity - Monte Carlo (2000 Trials)

Airfoil	Times Ranked #1	% of Trials	Avg Rank
airfoil_29	960	48.00	1.56
NACA_4412	803	40.15	4.02
refine_07	140	7.00	3.55
refine_13	87	4.35	2.59
refine_09	10	0.50	4.30
refine_01	0	0.00	9.14
refine_02	0	0.00	16.01
refine_03	0	0.00	7.88
refine_04	0	0.00	7.98
refine_05	0	0.00	11.89
refine_06	0	0.00	15.00
refine_08	0	0.00	16.73
refine_10	0	0.00	8.13
refine_11	0	0.00	9.18
refine_12	0	0.00	7.89
refine_14	0	0.00	13.28
refine_15	0	0.00	13.87

Table 8: Final combined Monte Carlo sensitivity results (2000 trials).

Final Combined One-at-a-Time (OAT) Weight Sensitivity Sweep (excerpt)

Criterion Varied	Weight Value	Winning Airfoil	TOPSIS Score
Mean C_L	0.05	refine_13	0.7390
Mean C_L	0.10	refine_13	0.7184
Mean C_L	0.15	refine_13	0.6863
Mean C_L	0.20	airfoil_29	0.6478
Mean C_L	0.25	airfoil_29	0.6143
Mean C_L	0.30	NACA_4412	0.6303
Mean C_L	0.35	NACA_4412	0.6637
Mean C_L	0.40	NACA_4412	0.6978
Mean C_D	0.05	NACA_4412	0.7770
Mean C_D	0.10	NACA_4412	0.7270
Mean C_D	0.15	NACA_4412	0.6639
Mean C_D	0.20	airfoil_29	0.6143
Mean C_D	0.25	airfoil_29	0.6316
Mean C_D	0.30	refine_07	0.6728
Mean L/D	0.30	refine_09	0.5766
Mean L/D	0.35	airfoil_29	0.5817
Mean L/D	0.40	airfoil_29	0.5967
Mean L/D	0.45	airfoil_29	0.6143
Mean L/D	0.50	airfoil_29	0.6339
Mean L/D	0.55	NACA_4412	0.6580
Mean CV	0.05	airfoil_29	0.6213
Mean CV	0.10	airfoil_29	0.6143
Mean CV	0.15	airfoil_29	0.6020
Mean CV	0.20	refine_09	0.5988

Table 9: Final combined one-at-a-time (OAT) weight sensitivity sweep.

This section is necessary as it assesses whether the optimization produced a genuine improvement over the baseline. Hence, the combined TOPSIS of the 15 refined airfoils along with airfoil_29 and NACA 4412 was conducted to see if refine_13 or airfoil_29 produces a higher TOPSIS score than its predecessor (s) from whom it became the optimum.

This, surprisingly, produces an interesting finding: looking at the TOPSIS and sensitivity results in Tables 7 and 8 respectively, airfoil_29 ranked first, with a narrow margin over the other airfoils. A sensitivity test was conducted for safety in case any results changed there, but the sensitivity test also presented airfoil_29 as the clear winner of the round. The sensitivity rankings suggest reasonable robustness at the stated criteria weights, remaining consistent with the nominal TOPSIS result; however, as the following analysis shows, certain individual criteria carry greater sensitivity than others. Examining the OAT results in detail:

- For mean C_L , the top-ranked airfoil shifts as its weight increases: refine_13 ranks highest at low weights (0.05–0.15), airfoil_29 at moderate weights (0.20–0.25), and NACA 4412 at higher weights (0.30–0.40). This indicates that NACA 4412 achieves the highest mean C_L among the candidates, while refine_13 achieves the lowest.
- For mean C_D , NACA 4412 ranks highest at low weights (0.05–0.15), airfoil_29 at moderate weights (0.20–0.25), and refine_07 at the highest tested weight (0.30). Since C_D is treated as a cost criterion, this indicates that refine_07 achieves the lowest mean drag among the candidates, while NACA 4412 achieves the highest.
- For mean L/D – the primary ranking criterion – refine_09 ranks highest at the lowest tested weight (0.30), airfoil_29 across the intermediate range including the nominal weight (0.35–0.50), and NACA 4412 at the highest tested weight (0.55).
- For the coefficient of variation (CV) of L/D , airfoil_29 ranks highest at low-to-moderate weights (0.05–0.15), including the nominal weight, while refine_09 ranks highest at the highest tested weight (0.20).

5.4.1 refine_13 vs NACA 4412

Per-Reynolds-Number L/D Comparison: refine_13 vs NACA4412

Reynolds Number	refine_13 L/D	NACA4412 L/D	Difference (%)	Better Performer
100,000	43.265	43.078	+0.44%	refine_13
300,000	75.778	73.233	+3.48%	refine_13
500,000	86.295	89.700	-3.80%	NACA4412
1,000,000	102.771	104.410	-1.57%	NACA4412
2,000,000	118.607	121.220	-2.16%	NACA4412

Table 10: Per-Reynolds-number L/D comparison: refine_13 vs NACA 4412.

As this is a paper on a range of Re, it is only fair to analyze each airfoil at each varied Re before presenting one for the range. From Table 10, we can see that refine_13 performs better at lower Re and NACA 4412 performs better at higher Re (indicated by the values of the main performance metrics, L/D ratio). This confirms that performance trends differ meaningfully between low and high Reynolds number regimes, though this per-Reynolds-number variation is distinct from the pooled TOPSIS-based comparison presented above.

5.4.2 refine_13 vs airfoil_29

airfoil_29 vs refine_13: Pooled Performance Comparison

Metric	airfoil_29	refine_13	Difference
Mean CL	1.0832	1.0757	-0.7%
Mean CD	0.01524	0.01519	-0.3%
Mean L/D	85.166	85.343	+0.2%
CV (%)	33.46	33.52	+0.2%

Table 11: Pooled performance comparison: airfoil_29 vs refine_13.

airfoil_29 vs refine_13: Per-Reynolds-Number L/D Comparison

Reynolds Number	airfoil_29 L/D	refine_13 L/D	Difference (%)	Better Performer
100,000	42.939	43.265	+0.76%	refine_13
300,000	76.025	75.778	-0.32%	airfoil_29
500,000	85.479	86.295	+0.95%	refine_13
1,000,000	104.431	102.771	-1.59%	airfoil_29
2,000,000	116.956	118.607	+1.41%	refine_13

Table 12: Per-Reynolds-number L/D comparison: airfoil_29 vs refine_13.

From Table 11, we can see that refine_13 has a lower mean Cd value (0.01519) and a higher mean L/D ratio (85.343), airfoil_29 has a higher Mean C_L (1.0832) and a lower CV value (33.46) (%). Since the differences in the values are small (maximum difference is 0.7%), we would be saying that airfoil_29 is better as it has a higher percentage difference in the metrics that it won compared to refine_13

However, this crossover does not show a clear trend like the one with refine_13 and NACA 4412, hence, cannot be used to determine the better airfoil between airfoil_29 and refine_13. Table 12 represents a per-Re crossover, where refine_13 and airfoil_29 alternate with Re for airfoil performance (L/D). This crossover pattern differs from the one observed between refine_13 and NACA 4412, where a consistent trend was found across Reynolds number. The near-tie between refine_13 and airfoil_29 is consistent with how refine_13 was generated: it results from only a 5% perturbation to airfoil_29's

design variables (Table 14), compared to the much larger design-variable difference between airfoil_29 and NACA 4412. This smaller geometric difference likely explains why refine_13 and airfoil_29 track each other closely across Reynolds numbers rather than diverging the way airfoil_29 and NACA 4412 do. This can also validate the result and performance for the lower change difference (5%).

5.4.3 airfoil_29 vs NACA 4412

Per-Reynolds-Number L/D Comparison: airfoil_29 vs NACA4412

Reynolds Number	airfoil_29 L/D	NACA4412 L/D	Difference (%)	Better Performer
100,000	42.939	43.078	-0.32%	NACA4412
300,000	76.025	73.233	+3.81%	airfoil_29
500,000	85.479	89.700	-4.71%	NACA4412
1,000,000	104.431	104.410	+0.02%	airfoil_29
2,000,000	116.956	121.220	-3.52%	NACA4412

Table 13: Per-Reynolds-number L/D comparison: airfoil_29 vs NACA 4412.

From the Table 13 we can see that performance across the range of Reynolds numbers alternates between airfoil_29 and NACA 4412; at Re 100000, 500000 and 2000000, NACA 4412 performs better while at Re 300000 and 1000000, airfoil_29 performs better.

This also doesn't produce a clear pattern in the overall better performance. The better airfoil cannot be found based on the difference % also as it varies for each Reynolds number and does not produce a consistent pattern. This table, hence, is useful to observe the performance at each tested Reynolds number, providing possible replacements in real life flight performances if needed.

5.5 Airfoil geometry comparison

5.5.1 refine_13 vs NACA 4412

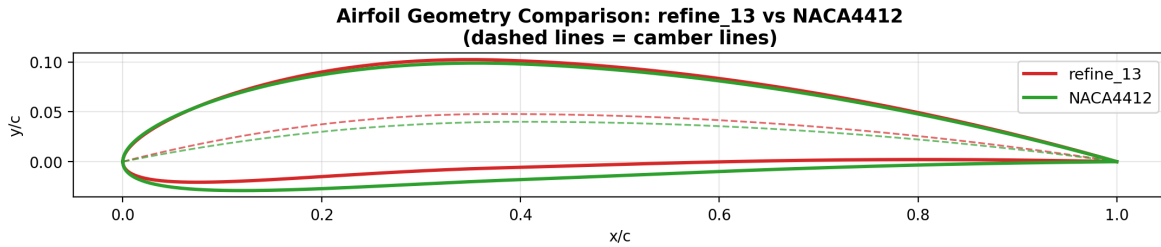


Figure 4: Geometric comparison of refine_13 and NACA 4412.

From Figure 4, we can see that refine_13 has a higher camber but is thinner than NACA 4412, with a slightly more forward camber position (also indicated by the difference in y/c coordinates). This is consistent with thin-airfoil theory: a higher camber increases lift, while a thinner airfoil reduces drag [12]. This helps explain refine_13's improved lift-to-drag ratio relative to NACA 4412 at lower Reynolds numbers, though this advantage diminishes and reverses at higher Reynolds numbers.

5.5.2 refine_13 vs airfoil_29

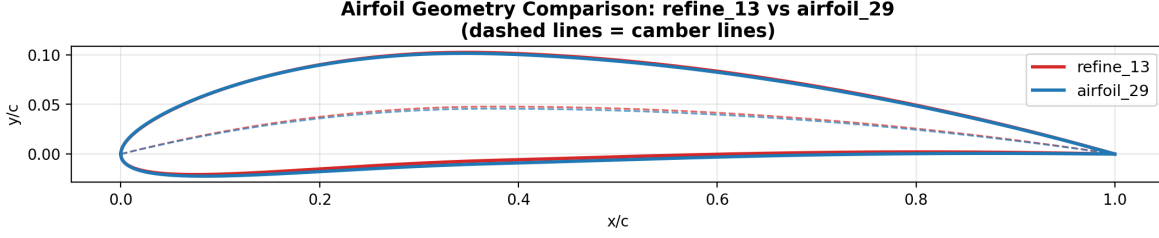


Figure 5: Geometric comparison of refine_13 and airfoil_29.

From Figure 5, we can see that the shape of refine_13 and of airfoil_29 are close to each other. According to Thin airfoil theory, refine_13 is supposed to have a higher L/D ratio as it is thinner and has a higher camber, but in this case, even with a thicker frame and a lower camber position, airfoil_29 still produces a better overall (TOPSIS-weighted) performance, despite refine_13 leading on L/D and CD individually. The 2 cambers follow fairly similar trajectories, indicating that the second round optimization did not produce that much of an effect on the airfoil performance than the first round. This indicates that airfoil_29 already contains a high performing metrics, as the optimum from the 2nd round (refine_13) is not that different - in terms of shape - compared to airfoil_29.

Physically, this implies that airfoil_29 and refine_13 produce similar pressure gradients to produce very similar lift at low Reynolds numbers (refer to Table 12), however, at higher Reynolds numbers, the performance gradually changes (+1%).

5.5.3 airfoil_29 vs NACA 4412

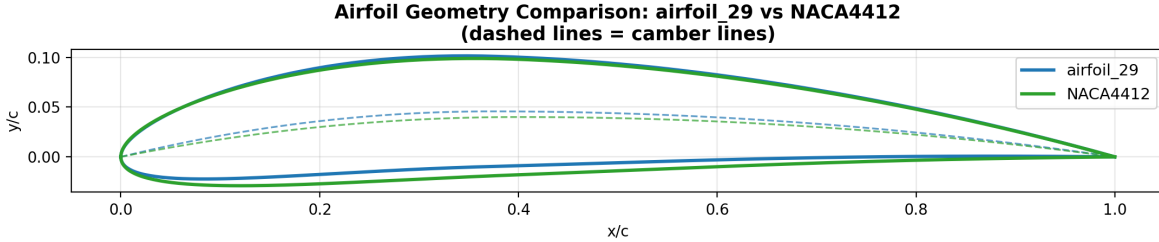


Figure 6: Geometric comparison of airfoil_29 and NACA 4412.

From Figure 6, we can observe that airfoil_29 is thinner, has higher camber, and a more forward camber position than NACA 4412. The better performance of airfoil_29 over NACA 4412 can be observed through this geometric comparison: thin airfoil theory suggests that increased camber increases lift (higher Lift to drag ratio and C_L) and a thinner airfoil reduces drag (lower C_d) [12]. This hence explains airfoil_29's improved lift-to-drag ratio over its baseline NACA 4412, this cannot be attributed to Reynolds numbers as the Re crossover does not produce consistent and an obvious pattern in lift to drag ratio across the 5 tested Reynolds numbers.

5.6 Summary of results

Based on all optimization and evaluation rounds, airfoil_29 was selected as the optimized airfoil, its design variable combinations are:

airfoil_29: Final Design Variables

Design Variable	Value
Max Thickness (%)	11.28
Max Camber (%)	4.569043
Camber Position (%)	37.2132
Thickness Position (%)	30.00

Table 14: airfoil_29 final design variables

It is the optimum when you are considering Reynolds numbers 100000, 300000, 500000, 1000000 and 2000000 as well as an angle sweep from 0° to 12° in 2° increments. For other conditions, there may be a different combination of design variables which will be the optimum, but for my investigation, airfoil_29 is the final optimized airfoil.

This does not mean that the 2nd round of optimization was unnecessary; the 2nd round was required to see if there are any close combinations to airfoil_29 that can perform better than it, but the results present airfoil_29 as the optimum. We could actually gain lots of information from the 2nd round:

- the 5% combination change proved to be efficient as performances were close.
- airfoil_29 is the overall winner, though the 2nd round had airfoil combinations that performed better than airfoil_29 (refine_13, refine_07) in a few categories, but airfoil_29 performed better overall.
- It allowed us to observe how slight changes in combinations can affect performances: slight changes to the optimum and to the combination of airfoil_29 has produced a range of TOPSIS scores and sensitivities, refine_08, a slight change from airfoil_29, seemed to perform the worst (considerably worse than the airfoil_29 itself), this shows that performance can vary a lot even with the slightest changes, hence engineers need to be careful when optimizing the airfoils and must consider as many combinations as possible to correctly determine the best performing airfoil

5.7 Sensitivity and robustness discussion

We can be confident that airfoil_29 is the optimized and best-performing airfoil, based on the following criteria used to evaluate the data:

- TOPSIS - is a common ranking system [10] that is used to determine the best performing across a set of data or initial variables
- sensitivity test - Monte Carlo is used to determine whether an airfoil is optimum across many weights, and OAT also presents the best performing airfoil across each weight distribution. The results for these distributions have been discussed above and presented airfoil_29 as the better airfoil. Sensitivity provides more accuracy for TOPSIS as TOPSIS only contains the fixed weights, but by evaluating multiple weights combinations, sensitivity provides whether the airfoil is optimum if we change the weights slightly.

When score vs sensitivity overlap, the TOPSIS score is considered more important as it is the direct comparison of performance for the presented weights. Sensitivity shows the better performing airfoil if

the weights change, but since the weights are defined and explained for this paper, the performance with the current weights are taken priority than performances with different weights.

6 Conclusion

Therefore, while a single airfoil can be optimized to achieve improved overall aerodynamic performance across the investigated range of Reynolds numbers, no single design was found to consistently outperform all other airfoils at every Reynolds number. This indicates that a robust multi-Reynolds-number optimum is achievable, but performance remains dependent on the operating regime.

The objective of the experiment was to produce an optimized airfoil that has robust performance over a range of Reynolds numbers.

The final optimized airfoil was selected to be airfoil_29 with parameters previously established in Table 14

It was selected as a result of being the best performing airfoil in the TOPSIS, OAT and Monte Carlo sensitivity test with the 15 refine airfoils, NACA 4412 and the round 2 optimum (refine_13). airfoil_29 outperformed the baseline NACA 4412 under the stated TOPSIS criteria and weights.

The best performing airfoil was one from the first round optimization method; from this we can see that the 2nd round optimum failed to outperform the first round optimum. We can then evaluate the design space to suggest that a strong optimum was already found on the first optimization round, the 2nd round was required for precautionary measures in case any change occurred in optimum design variable combinations

The TOPSIS analysis on a few occasions - 2nd round optimum - produced close and negligible scores. To determine which airfoil is the optimum, a sensitivity test was performed to test the robustness of the results. The OAT used different combinations of weights and outputted the optimum airfoil if the weights were slightly changed from the given ones.

In terms of physical meaning, we cannot really make a comparable statement because if we see Table 13, we can see that the performances across the ranges of Reynolds numbers are not close other than at 100000 and 1000000, where there is less than a 0.5% difference, the rest show differences of approximately 3.5-4.7%. So from this we are to conclude that these two airfoils perform differently at different Reynolds numbers. However, per-Reynolds-number performance does not follow a simple low-Re/high-Re split: as shown in Table 13, NACA 4412 achieves the higher L/D at three of the five tested Reynolds numbers (100,000, 500,000, and 2,000,000), while airfoil_29 achieves the higher L/D at the remaining two (300,000 and 1,000,000). This is not the main optimization detection criteria but just a distribution of performance for the comparison of the physical implications of the airfoil design variable combinations. This may contradict our objective function, but the optimized airfoil indicated that it is better as an average across all the tested Reynolds numbers. Here, NACA 4412 and airfoil_29 can practically be interchanged for different Reynolds numbers' flights, but across all experimental Reynolds numbers, if one airfoil had to be constructed: the results favor airfoil_29 over NACA 4412.

7 Acknowledgments

The author thanks the developers of XFOIL (M. Drela, MIT) and XFLR5, whose software was used for the aerodynamic analyses. The computational workflow was implemented in Python. Generative AI tools assisted in writing and debugging parts of the analysis code, and in editing the manuscript's language and style. The author reviewed, tested and validated all outputs, verified the accuracy of the text, and takes full responsibility for the content of this work. This research received no external funding.

Limitation:

- Airfoil_18 was not present in the tests because there was an issue with the automatic generation of the codes in XFOIL. But due to the pattern in airfoil performance, airfoil_18 would not be close to the optimum as its camber region places it along with the cambers of the 5 lowest performing airfoils.
- The angle-of-attack sweep spanned 0° to 12° in 2° increments, yielding 7 data points, rather than the 13 points that a 1° increment would have produced.
- There were only 5 Reynolds number ranges that covered the low, medium and high Reynolds numbers ranges. With only a limited number of variable values, the results may not be as accurate than if a complete test involving many variable values were involved (taking each Reynolds number, each angle)
- The results were computed entirely using XFOIL’s analysis methods, with no higher-fidelity CFD simulations or wind-tunnel testing conducted to validate the results. This limits the ability to confirm the absolute accuracy of the predicted aerodynamic performance and introduces potential uncertainty into the final results.
- XFOIL’s boundary-layer formulation is known to lose accuracy at Reynolds numbers below approximately $2\text{--}3 \times 10^5$ [16], where laminar separation bubbles form and transition prediction becomes increasingly unreliable. Since the lowest Reynolds number tested in this study was 1×10^5 , the results at this condition — and potentially at 3×10^5 — should be interpreted with reduced confidence relative to the higher-Re cases.
- This study’s design space was restricted to the NACA 4-digit parametric family, varying only maximum thickness, maximum camber, and camber position. This parametrization cannot represent airfoil shapes outside this family, such as those achievable through more flexible geometric representations (e.g. NURBS or CST parametrization) used in other multi-point optimization studies [17]. The reported optimum is therefore the best airfoil within the NACA 4-digit design space, not a global optimum across all possible airfoil shapes.
- The TOPSIS scores of the first- and second-round optimizations differed only by small margins, requiring a predefined priority rule to determine the preferred design. However, no uncertainty or noise analysis was conducted on the airfoils with closely comparable scores, potentially limiting the confidence and accuracy of the final ranking.
- The exact Monte Carlo perturbation percentage was not recorded, so the reported sensitivity results cannot be reproduced exactly.
- The automated pipeline and XFLR5 differ by about 6% on average (up to 8.5%), so absolute performance values are tool-dependent and conclusions rely on relative rankings.

Future work could include an increase in objective and design variable values, multiple runs and using many of the standard NACA airfoils. LHS, DOE can have thousands of runs, this is to ensure that an airfoil closest to optimum can be found at all Reynolds numbers. Works on structural conditions, finite wings, fuel and production cost, the optimum required material, optimum conditions for best performance. An airfoil’s applications elsewhere in the field of aerodynamics, like if the optimized airfoil can be used for wind turbines, etc.

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