COPS AND ROBBERS ON PERIOD-2 TEMPORAL PATHS AND CYCLES: MINIMUM VISIBILITY RADIUS FOR GUARANTEED CAPTURE

VINAMRA DHOOT

Abstract

Cops and Robbers is a pursuit-evasion game that traditionally assumes that the game is played on static graphs and there is no informational asymmetry. This paper studies the interaction between two relaxations of the assumptions: the graph is played on a periodic temporal graph (period-2, specifically) and the cop has an arbitrary visibility radius ℓ. The paper, first, proves that regardless of any number of vertices n, any visibility radius ℓ, and any period where every edge appears at least once per period, a path is cop win.

Then, the paper transitions to cycles. For cycles, we introduce a phase-change count $q(s)$, which measures how often adjacent edges belong to different phases. C_3 is cop-win exactly when either $q(s) = 2$ and $\ell \geq 0$, or $q(s) = 0$ and $\ell \geq 1$. We also prove that for every $n \geq 4$, a cycle C_n is always robber-win when the visibility radius $\ell = 0$. For C_4, the cycle is cop-win exactly when $q(s) = 2$ and $\ell \geq 1$. C_5 is cop-win exactly when either $q(s) = 4$ and $\ell \geq 1$ or $q(s) = 2$ and $\ell \geq 2$. For C_6 and C_7, we prove through exhaustion that C_6 is cop-win when the visibility radius is $\ell \geq 2$ and the schedule is either 000101 or 001011, up to rotation, reflection, and phase interchange; and C_7 is cop-win for a visibility radius $\ell \geq 3$ and $q(s) = 6$. Existing results then allow us to show that all C_n, where $n \geq 8$, is robber-win.

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1. Introduction

Cops and robbers is a classic *pursuit-evasion* game played on a graph, where the cop tries to occupy the same vertex as the robber while the robber tries to avoid capture indefinitely. Aigner and Fromme [AF84] establish the rules for the game. The cop occupies the vertex first followed by the robber. On each turn, the players can either move to a nearby vertex or stay on the same vertex. Both the cop and robber are assumed to play optimally, therefore, the cop always tries to find a winning strategy while the robber always tries to find an evasion strategy. The game takes place on a static graph with no informational asymmetry. Graphs where the cop has a winning strategy are termed as cop-wins while the graphs where the robber can *evade* indefinitely are known as robber-win graphs.

The two major assumptions the original game largely relies on are: (a) the graph is static, and (b) there is no informational asymmetry. However, two separate strands of work relax these assumptions. Clarke et al. [CCD$^+$17] study limited visibility Cops and Robbers, where the cop only sees the robber when the distance is at a visibility radius ℓ. Separately, De Carufel et al. [CFSS24] study Cops and Robbers on periodic temporal graphs, where the active edge set changes over time repeatedly on a sequence of snapshots.

Related work by Erlebach and Spooner [ES19] considers edge-periodic, where individual edges follow repeating binary activity patterns.

While limited visibility and temporal graphs have been studied independently, their interaction changes both the movement structure of the game and the information available for a cop. This raises the question: how does limited visibility affect the outcome of Cops and Robbers if the graph changes periodically. This is first looked at on paths and then on cycles, where the absence of endpoints make the pursuit substantially different. Even in the classical game, paths and cycles differ significantly.

Therefore, this paper looks at the interaction of these two restrictions: periodic changes in edge availability and incomplete positional information. We focus on one cop and robber, period-2, and an arbitrary visibility radius ℓ. We introduce a phase change count $q(s)$ for cycles to differentiate the temporal edge arrangements. This helps us see if there is a correlation between the value of $q(s)$ and the number of vertices in the cycle n. The analysis shows that there is a significant difference between paths and cycles and recognizes a universal robber-win behavior of a cycle for a sufficiently large n.

1.1. Methodology

We use analytical proofs and computational assistance. We create a sweeping strategy for paths. For cycles C_3, C_4, C_5, we prove cop and robber-wins using analytical proofs, which incorporate cop and robber strategies and belief sets. For C_6 and C_7, we exhaustively prove winning strategies using an exact belief-state solver (in Appendix A.7). For $n \geq 8$, we use Erlebach and Spooner's result [ES19].

The paper is structured as follows: section 2 defines the rules of the game. Then, section 3 defines the important terms that are used recurrently in this paper. Followed by this, section 4 looks at paths. Section 5 looks at cycles, primarily seeing how C_3, C_4, C_5, and C_6 and beyond differ from each other. Lastly, section 6 concludes.

2. Rules of the Game

The temporal schedule s is defined before the game is played and both the players know s in advance. The initial phase is $\tau_0 = 0$. However, this is not restrictive as we prove later that there is phase interchange symmetry in section 5.3.

The cop chooses the starting vertex c_0 first, followed by the robber $r_0 \neq c_0$. After both players have chosen their starting vertex, the cop receives an initial visibility observation. The cop knows the robber's exact location whenever $d_G(c_t, r_t) \leq \ell$. Otherwise, the observation is "invisible", although the cop may infer the robber's position from the observation history. The robber remains invisible and the cop has to reason through their belief set B_t (Definition 3.11). The cop moves first in every round and the players can only move on an active edge or stay at the same position. However, the robber has full information about the cop's position and the limited visibility restriction only applies to the cop.

Capture is checked after every move made by the players. If the cop lands on the same vertex as the robber's current vertex, the game ends immediately. After both players have moved, the cop receives a visibility observation at the new position. After both players make a move, the phase changes $\tau_{t+1} = 1 - \tau_t$.

If the cop has a deterministic cop strategy guaranteeing capture in finitely many rounds against every legal robber move, the graph is cop-win. However, if the robber is able to evade capture indefinitely, then the graph is robber-win.

3. Definitions

Definition 3.1. Let $G = (V, E)$ be a finite, simple, undirected graph. The *path graph* P_n has vertex set

$$V = \{v_1, \ldots, v_n\},$$

and edge set

$$E = \{\{v_i, v_{i+1}\} : 1 \leq i < n\}.$$

Definition 3.2. We define a *cycle* C_n as:

$$C_n = v_1 - v_2 - \cdots - v_n - v_1,$$

with edges $e_1, \ldots, e_n$.

Definition 3.3. For vertices u, v in a connected graph G, the *distance* $d_G(u, v)$ is the minimum number of edges in a path joining u to v.

For the path $P_n = v_1 - \cdots - v_n$,

$$d_{P_n}(v_i, v_j) = |i - j|.$$

For the cycle $C_n = v_1 - \cdots - v_n - v_1$,

$$d_{C_n}(v_i, v_j) = \min\{|i-j|,\ n-|i-j|\}.$$

Throughout this paper, visibility is measured in the underlying graph.

Definition 3.4. A *temporal graph* $\mathcal{G}$ is defined as:

$$\mathcal{G} = (G_0, G_1, \dots),$$

where every

$$G_t = (V, E_t),$$

where they have the same V but potentially a different E_t [CFSS24].

Definition 3.5. A temporal graph has a *period* p if,

$$G_{t+p} = G_t,$$

for every $t \geq 0$.

This paper specifically focuses on a **period 2** temporal graph.

This means

$$\mathcal{G} = (G_0, G_1)^*.$$

Definition 3.6. The *temporal phase* at round t is defined by

$$\tau_t = t \bmod 2.$$

The active snapshot at round t is therefore, G_{τ_t}.

Definition 3.7. Let G be a path P_n or a cycle C_n, with vertices $v_1, \dots, v_n$. Label the edges by

$$e_i = \{v_i, v_{i+1}\}, \qquad 1 \leq i < n.$$

For C_n, additionally define $e_n = \{v_n, v_1\}$.

The *period-two edge schedule* is a binary word

$$s = \begin{cases} (s_1, \dots, s_{n-1}) \in \{0,1\}^{n-1}, & G = P_n, \\ (s_1, \dots, s_n) \in \{0,1\}^n, & G = C_n, \end{cases}$$

where s_i is the phase in which edge e_i is active.

Definition 3.8. The *closed neighborhood* of a vertex v in snapshot G_t is defined as

$$N_{G_t}[v] = \{v\} \cup \{u \in V : \{u, v\} \in E_t\}.$$

Definition 3.9. A *cop's position* is defined as:

$$c_t \in V,$$

and a *robber's position* is defined as:

$$r_t \in V.$$

This game has only one cop and robber. Like the game, the cop chooses a vertex first followed by the robber.

Definition 3.10. Let $\ell \in \mathbb{Z}_{\geq 0}$ be the cop's *visibility radius*. The visible set from a vertex c is [CCD+17]

$$V_\ell(c) = \{v \in V : d_G(c, v) \leq \ell\},$$

where d_G denotes distance in the underlying graph G.

At each observation time t, the robber is visible if and only if

$$r_t \in V_\ell(c_t).$$

Definition 3.11. We define the *belief set* as

$$B_t \subseteq V,$$

to be the set of all robber positions that is consistent with the cop's observations up until time t.

For a set of possible positions $X \subseteq V$, define

$$M_\tau(X) = \bigcup_{v \in X} N_{G_\tau}[v].$$

Thus, $M_\tau(X)$ consists of all vertices reachable in one legal move from some vertex in X during phase τ, including by staying.

Definition 3.12. For a vertex $v \in V(G)$ in a cycle C_n, where n is the number of vertices in the cycle, its *antipodal set* is defined as:

$$A_i := A_v = \{u \in V(G) : d_G(u, v) = \mathrm{ecc}_G(v)\},$$

where $\mathrm{ecc}_G(v)$ is the eccentricity of v and is defined as

$$\mathrm{ecc}_G(v) = \max_{x \in V(G)} d_G(v, x).$$

4. Paths

Theorem 4.1. *Let $\mathcal{G}$ be a periodic temporal graph whose footprint is the path:*

$$P_n = v_1, v_2, \ldots, v_n,$$

and suppose every edge is active at least once during each period. Then, one cop can guarantee the capture of the robber.

Proof. Consider the set:

$$R_1 = \{v_2, \ldots, v_n\},$$

which is the set of all possible positions of the robber when the cop is at v_1.

Suppose the cop is at v_i then the robber must be in the set:

$$R_i = \{v_{i+1}, \ldots, v_n\}.$$

Since the cop occupies the region v_i, the robber cannot move to the cleared region before that without passing through v_i and hence, being captured.

When the cop moves from v_i to v_{i+1}, if the robber is at v_{i+1} capture occurs. If the robber is not at v_{i+1}, the set shrinks. This shrinking keeps happening:

$$R_1 \supset R_2 \supset R_3 \supset \cdots \supset R_n,$$

until the set:

$$R_n = \varnothing.$$

Hence, the cop has a winning strategy. Therefore, given that each edge is active at least once each period, the path will be a cop-win for any period $p \geq 1$ and every visibility radius $\ell \geq 0$. ■

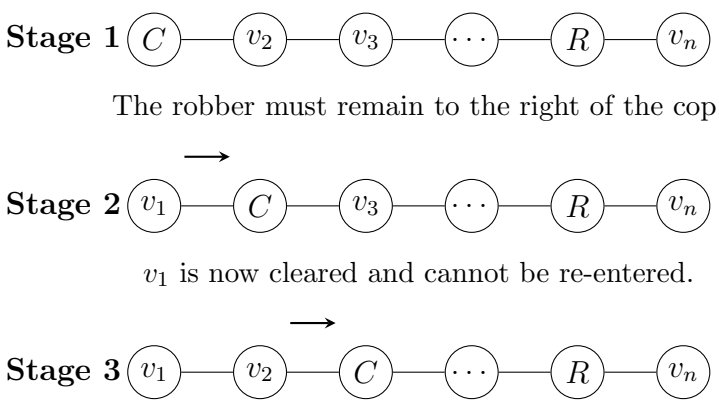

Figure 1. The sweeping strategy on a temporal path.

5. Cycles

On a path, the cop acts as a barrier that clears vertices. However, on a cycle the robber can go to the other side. This means the shrinking of the set:

$$R_1 \supset R_2 \supset R_3 \supset \cdots \supset R_n.$$

no longer exists. This is why we must analyze cycles separately.

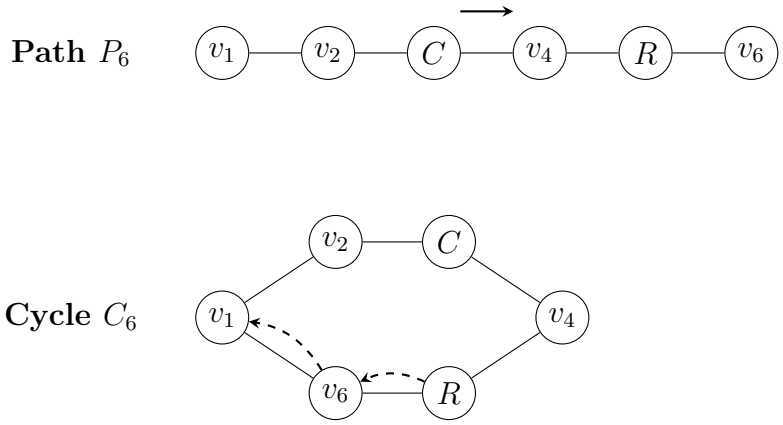

Figure 2. Difference between paths and cycles.

Figure 2 makes this comparison more intuitive.

Definition 5.1. We define a *phase-change count* as

$$q(s) = |\{i : s_i \neq s_{i+1}\}|, \ s_{n+1} = s_1,$$

as a measure of how often adjacent edges are in different temporal phases.

Lemma 5.2. *For every binary cyclic schedule $s \in \{0,1\}^n$, $q(s)$ is even. Moreover, for every even q:*

$$|\{s \in \{0,1\}^n : q(s) = q\}| = 2\binom{n}{q}.$$

Proof. For a binary schedule s for a cycle C_n, every time a phase changes from 0 to 1, it must also change back from 1 to 0 since the last vertex v_n and v_1 are connected. This means that phase changes occur in pairs. Hence, $q(s)$ is always even.

For n boundaries in a cycle, we can choose q of these n boundaries in $\binom{n}{q}$ ways. Since the first entry can be either 1 or 0, we multiply the entire binomial by 2. Hence, $2\binom{n}{q}$. □

5.1. Main Theorem

Theorem 5.3. *Let $n \geq 3$ and let $s \in \{0,1\}^n$ define an exclusive period-two temporal cycle $C_n(s)$.*

Write

$$q(s) = \big|\{i \in \{1,\dots,n\} : s_i \neq s_{i+1}\}\big|, \qquad s_{n+1} = s_1.$$

Let $\ell_*(C_n(s))$ be the smallest non-negative integer visibility radius at which one cop has a deterministic strategy guaranteeing capture, with $\ell_*(C_n(s)) = \infty$ if no such radius exists.

Then

$$\ell_*(C_n(s)) = \begin{cases} 0, & n = 3,\ q(s) = 2, \\ 1, & n = 3,\ q(s) = 0, \\ 1, & n = 4,\ q(s) = 2, \\ 1, & n = 5,\ q(s) = 4, \\ 2, & n = 5,\ q(s) = 2, \\ 2, & n = 6,\ s \sim 000101 \text{ or } s \sim 001011, \\ 3, & n = 7,\ q(s) = 6, \\ \infty, & \text{otherwise}, \end{cases}$$

where $\sim$ denotes equivalence under rotations, reflections, and interchange of the two phases.

Consequently, for every $\ell \in \mathbb{Z}_{\geq 0}$, the cycle $C_n(s)$ is cop-win if and only if $\ell \geq \ell_*(C_n(s))$.

The following subsections establish this classification. The cases $n \leq 5$ are proved analytically, while the cases $n = 6, 7$ are established by the exact belief-state solver. For $n \geq 8$, the conclusion follows from the full-visibility evasion theorem of Erlebach and Spooner [ES19].

5.2. Monotonicity in Visibility

Lemma 5.4. *For temporal graph $\mathcal{G}$, if a cop has a winning strategy with visibility radius ℓ_1, then the cop has a winning strategy with every visibility radius $\ell_2 \geq \ell_1$.*

Proof. Let σ be a winning strategy on $\mathcal{G}$ with visibility radius ℓ_1. Let $\ell_2 \geq \ell_1$. For every cop position c, $V_{\ell_1}(c) \subseteq V_{\ell_2}(c)$. Then, the cop can apply the same winning strategy σ by considering the robber to be invisible when $d(c_t, r_t) > \ell_1$. Since movement and capture rules

are unchanged, the game ends in capture. Hence, if $\mathcal{G}$ is cop-win at visibility radius ℓ_1, then $\mathcal{G}$ is also cop-win at visibility radius ℓ_2. □

5.3. Preliminary symmetries

First, rotation and reflection only relabel the cycle. Therefore, if a schedule s is cop-win then the reflected and rotated schedule is also cop-win.

Lemma 5.5. *Let s be a period-two schedule and let s' be obtained by interchanging 0 and 1. If s is cop-win, then s' is also cop-win and vice versa.*

Proof. Consider the snapshot of s to be $G_0, G_1, G_0, G_1, \ldots$. This means the snapshot of s' is $G_1, G_0, G_1, G_0, \ldots$. Since waiting is allowed in this game, in s', the cop can wait one round. This means that after waiting, the snapshot becomes $G_0, G_1, G_0, \ldots$, which is the same as s. Even if the robber moves, the winning strategy for s succeeds against all starting positions of the robber. Therefore, the new robber position r' can be considered as a new initial starting position, and the cop can apply its winning strategy for s. Applying the same reasoning, if the cop waits in the first round in s, then the new snapshot becomes the same as s'. Hence, any schedule remains cop-win under phase interchange. □

5.4. Cycle C_3

Theorem 5.6. *Let $s \in \{0,1\}^3$ be a period-two temporal triangle; then, if $q(s) = 2$, the cycle is cop-win $\forall\ \ell \geq 0$ and if $q(s) = 0$, the cycle is cop-win if and only if $\ell \geq 1$.*

Proof. We define $\ell_*(C_3(s))$ as the smallest visibility radius for a cycle to be cop-win.

First, we take $q(s) = 2$.

Take $\ell = 0$. Label each vertex as a, b, c. The vertices hold symmetry under phase interchange and rotation. This means the set of active edges at τ are $E_\tau = \{ab, bc\}$ and at $E_{1-\tau} = \{ac\}$. The cop starts at $c_0 = b$. The cop follows the legal sequence $b \to a \to c \to b$. The belief set at c_0 is $B_0 = \{a, c\}$. For a cop move c_{t+1}, the possible surviving positions for the robber are:

$$B_{t+1} = \left(\bigcup_{r \in B_t \setminus \{c_{t+1}\}} N_{G_{\tau_t}}[r] \right) \setminus \{c_{t+1}\} \tag{5.1}$$

Therefore, applying Equation 5.1 to the other belief sets, we get:

$$B_1 = N_{G_\tau}[c] \setminus \{a\} = \{c, b\},$$
$$B_2 = N_{G_{1-\tau}}[b] \setminus \{c\} = \{b\},$$

and

$$B_3 = \bigcup_{r \in \varnothing} N_{G_\tau}[r] = \varnothing.$$

Therefore, for $q(s) = 2$, $\ell_*(C_3(s)) = 0$.

Second, we take $q(s) = 0$.

For $q(s) = 0$, either all edges are active or they are all inactive. If $\ell = 1$ and the cop chooses any vertex $c_0 \in \{v_1, v_2, v_3\}$. Since visibility radius is 1, $V_\ell(c) = V \ \forall c \in V$. Therefore, for any robber position, the cop can capture the robber whenever the edges become active.

For $\ell = 0$, fix any deterministic cop strategy. Since the robber remains invisible before capture, the cop's moves along a non-capture history are determined. Let c_0 be the initial cop position and x_k the position after the k-th active-round cop move. The robber can always choose a vertex

$$r_k \in V \setminus \{x_k, x_{k+1}\},$$

since

$$|V \setminus \{x_k, x_{k+1}\}| \geq 1.$$

Thus, the initial choice r_0 avoids capture on the first cop move, and every subsequent choice is a legal robber response that avoids the cop's current position and next destination, because the active snapshot is K_3. This schedule simply adds "stay" rounds between active rounds, during which neither player moves. Hence, the robber evades capture indefinitely.

Hence, for $q(s) = 2, \ell_*(C_3(s)) = 0$ and for $q(s) = 0, \ell_*(C_3(s)) = 1$. ■

5.5. Zero Visibility

Lemma 5.7. *Let $n \geq 4$, and let $s \in \{0,1\}^n$ define a period-two temporal cycle $C_n(s)$. At visibility radius $\ell = 0$, no deterministic cop strategy guarantees capture.*

Proof. Fix any deterministic cop strategy. We define the belief set as:

$$B_{t+1} = \left(\bigcup_{r \in B_t \setminus \{c_{t+1}\}} N_{G_{\tau_t}}[r] \right) \setminus \{c_{t+1}\}. \tag{5.2}$$

We prove that the belief set always has one of the following forms:

$$\begin{aligned} &\text{(I)} && B_t = V \setminus \{c_t\}, \\ &\text{(II)} && B_t = V \setminus \{c_t, c_{t-1}\}, \qquad t \geq 1, \quad c_t \neq c_{t-1}. \end{aligned}$$

In case (II), the vertices c_t, c_{t-1} are adjacent. Writing y for the other neighbor of c_{t-1}, we additionally require

$$c_t c_{t-1} \notin E_{\tau_t}, \qquad c_{t-1} y \in E_{\tau_t}.$$

Initially, $B_0 = V \setminus \{c_0\}$, so (I) holds. Suppose the invariant holds at the beginning of round t.

Case I: $B_t = V \setminus \{c_t\}$.

If the cop stays, every other vertex remains possible because the robber may stay. Hence, $B_{t+1} = V \setminus \{c_t\}$, and (I) persists.

Otherwise, let $u = c_{t+1}$ be the cop's destination, and let v be the other neighbor of c_t. Immediately after the cop's move, the surviving positions are $V \setminus \{c_t, u\}$.

Each of these positions remains possible by staying. The vacated vertex c_t becomes possible exactly when vc_t is active. Therefore,

$$B_{t+1} = \begin{cases} V \setminus \{u\}, & vc_t \in E_{\tau_t}, \\ V \setminus \{u, c_t\}, & vc_t \notin E_{\tau_t}. \end{cases}$$

The first outcome satisfies (I). In the second outcome, $c_t u$ was active because the cop used it, while vc_t was inactive. Since each edge belongs to exactly one phase,

$$uc_t \notin E_{\tau_{t+1}}, \qquad c_t v \in E_{\tau_{t+1}}. \tag{5.3}$$

Thus (II), including its edge conditions, holds at round $t+1$.

Case II: $B_t = V \setminus \{c_t, c_{t-1}\}$.

By (5.3), the cop cannot move to c_{t-1} because that edge is inactive.

If the cop stays, a possible robber at y can move to c_{t-1}. Every other non-cop vertex remains possible by staying, so

$$B_{t+1} = V \setminus \{c_t\}.$$

Hence (I) holds.

Otherwise, the cop moves to their other neighbor $u = c_{t+1}$. Since $n \geq 4$, we have $y \neq u$. Thus a robber at y survives the cop's move and can enter c_{t-1}.

However, the vacated vertex c_t cannot be reached by a surviving robber. Its only neighbors are u, where a robber would have been captured, and c_{t-1}, which was not in B_t. Consequently, $B_{t+1} = V \setminus \{u, c_t\}$.

The edge $c_t u$ was active, while $c_t c_{t-1}$ was inactive. After the phase changes,

$$uc_t \notin E_{\tau_{t+1}}, \qquad c_t c_{t-1} \in E_{\tau_{t+1}}.$$

These are precisely the edge conditions required for (II) at round $t+1$.

Therefore, the invariant is preserved under every legal cop move. By induction,

$$|B_t| \in \{n-1, n-2\}, \qquad \text{and hence} \qquad |B_t| \geq n-2 \geq 2 \quad \forall t \geq 0.$$

At $\ell = 0$, every observation before capture is invisible. The nonempty beliefs therefore give capture-free robber histories of every finite length against the fixed cop strategy. These

histories form a finitely branching rooted tree. By König's infinity lemma, the tree contains an infinite path, corresponding to an infinite capture-free play.

Hence no deterministic cop strategy guarantees capture. □

5.6. Cycle C_4

Lemma 5.8. *Let $s \in \{0,1\}^4$ define a period-two temporal cycle $C_4(s)$ with $q(s) = 0$. Then the robber can evade capture indefinitely for every visibility radius $\ell \geq 0$.*

Proof. For the classical cops and robbers game, we know that for every cycle where $n \geq 4$, the cycle is a robber-win graph [AF84].

Therefore, when $q(s) = 0$,

$$s \in \{0000, 1111\}.$$

This means one phase is the ordinary C_4 cycle, while the other has no active edges. These empty phases simply keep adding waiting rounds. This means the robber strategy still works. Hence, $q(s) = 0$ is a robber-win graph. This lemma is true for all C_n where $n \geq 4$. □

Theorem 5.9. *Let $s \in \{0,1\}^4$ define a period-two temporal cycle $C_4(s)$. For every integer $\ell \geq 0$, one cop has a strategy guaranteeing capture if and only if $q(s) = 2 \quad$ and $\quad \ell \geq 1$.*

Proof. For $\ell = 0$, we use Lemma 5.7.

Set $\ell \geq 0$.

For $q(s) = 4$, opposite edges have the same phase. Let f be the opposite-vertex map. Then

$$uv \in E_\tau \iff f(u)f(v) \in E_\tau.$$

The robber starts at $r_0 = f(c_0)$ and responds to each cop move by choosing $r_{t+1} = f(c_{t+1})$. This response is legal because opposite edges are active simultaneously; if the cop stays, the robber stays. Before each cop move the players are distance 2 apart, preventing immediate capture, and the robber's response restores this distance. Therefore, the robber evades indefinitely for every $\ell \geq 0$.

Set $\ell = 1$:

For $s = 0011$:

$$E_0 = \{e_1, e_2\} \text{ and } E_1 = \{e_3, e_4\}.$$

Choose, $c_0 = v_2$.

Then, $r_0 = v_4$, to avoid capture on the first move because if the robber chooses $r_0 = v_1$ or v_3, then both edges e_1 and e_2 are active, allowing immediate capture.

We can map this as the following: $(v_2, v_4) \to (v_3, v_4) \to (v_4, v_4)$.

Hence, this is a cop-win.

For $s = 0001$**:**

$$E_0 = \{e_1, e_2, e_3\} \text{ and } E_1 = \{e_4\}.$$

Again, choose $c_0 = v_2$ and $r_0 = v_4$ to avoid capture on the first move.

The cop moves from v_2 to v_1. The robber has two legal responses in $\tau = 0$. If the robber stays, then in the next move when $\tau = 1$, the edge e_4 becomes active allowing the cop to capture. If the robber moves from v_4 to v_3, then the robber is invisible from v_1 since $d(v_1, v_3) = 2 > \ell$. Therefore, this gives the cop information and the cop can infer the robber's position as v_3. This makes the belief set $B = \{v_3\}$. In phase 1, the cop moves from $v_1 \to v_4$, while the robber has to stay. When the phase changes back to $\tau = 0$, the cop moves $v_4 \to v_3$ and captures the robber immediately. It suffices to prove the cop-win cases at $\ell = 1$. Visibility monotonicity then gives the result for every $\ell \geq 1$.

Therefore, $s = 0001$ is cop-win for every $\ell \geq 1$.

Hence, $q(s) = 2$ is cop-win only when $\ell \geq 1$. ■

5.7. Cycle C_5

Lemma 5.10. *No deterministic cop strategy guarantees capture on $C_5(s)$, where $q(s) = 2$ and $\ell = 1$.*

Proof. $s = 00011$ and 00001, which up to rotation, reflection, and phase interchange is representative of all $q(s) = 2$.

First, take $s = 00011$.

Label the edges of the cycle C_5 as $a - b - c - d - e - a$, with

$$E_0 = \{ab, bc, cd\} \qquad E_1 = \{de, ea\}.$$

Fix any deterministic cop strategy. Along a history in which every observation is invisible, the belief set satisfies

$$B_0 = A(c_0), \qquad B_{t+1} = A(c_{t+1}) \cap \bigcup_{r \in B_t} N_{G_{\tau_t}}[r]. \tag{5.4}$$

Every position in B_t is distance 2 from c_t, so none is captured on the cop's next move. The update retains precisely the robber responses that remain invisible.

Suppose, for a contradiction, that T is the first round for which

$$B_T \neq \varnothing, \qquad B_{T+1} = \varnothing.$$

Thus, no robber response in round T leaves the robber invisible.

If $B_T = A(c_T)$, then for every legal cop destination,

$$A(c_T) \cap A(c_{T+1}) \neq \varnothing.$$

The antipodal sets of adjacent vertices on C_5 share one vertex. If the cop stays, the sets coincide. A robber at a shared vertex can stay invisible. Consequently, $B_{T+1} \neq \varnothing$, is a contradiction.

Since $A(c_T)$ contains exactly two vertices, B_T must therefore be a singleton.

For a singleton belief to become empty, the cop must move adjacent to the robber, and the robber's other incident edge, permitting escape away from the cop, must be inactive. Looking at E_0 and E_1, using the reflection

$$a \leftrightarrow d, \qquad b \leftrightarrow c, \qquad e \leftrightarrow e,$$

reduces the possible failure configurations to

$$\begin{aligned} &\tau_T = 0, \quad c_T = b, \quad B_T \in \big\{\{d\},\{e\}\big\};\\ &\tau_T = 1, \quad c_T = e, \quad B_T \in \big\{\{b\},\{c\}\big\};\\ &\tau_T = 1, \quad c_T = a, \quad B_T = \{d\}. \end{aligned} \tag{5.5}$$

We show that none can occur.

First, whenever the cop is at b in phase 0 at a round $t \leq T$,

$$B_t = \{d, e\}. \tag{5.6}$$

This holds initially if $c_0 = b$. At any later such round, the preceding phase is 1, in which b is isolated. Therefore, the cop stayed at b, and the preceding nonempty belief was a subset of $A(b) = \{d, e\}$.

Since edge de is active,

$$A(b) \cap N_{G_1}[d] = A(b) \cap N_{G_1}[e] = \{d, e\}.$$

Equation (5.4) therefore gives (5.6), excluding the first configuration in (5.5).

Second, whenever the cop is at e in phase 1 at a round $t \leq T$,

$$B_t = \{b, c\}.$$

The preceding phase is 0, in which e is isolated, so the cop stayed there. The preceding nonempty belief was contained in $A(e) = \{b, c\}$. Since bc is active,

$$A(e) \cap N_{G_0}[b] = A(e) \cap N_{G_0}[c] = \{b, c\}.$$

Thus $B_t = \{b, c\}$, excluding the second configuration.

Third, suppose the cop is at a in phase 1 at a round $t \leq T$. In the preceding phase 0, the cop either stayed at a or moved $b \to a$.

If the cop stayed, the preceding nonempty belief was contained in $\{c, d\}$. Since cd is active,

$$A(a) \cap N_{G_0}[c] = A(a) \cap N_{G_0}[d] = \{c, d\}.$$

If the cop moved from b, equation (5.6) gives the preceding belief as $\{d, e\}$. Hence,

$$B_t = A(a) \cap \big(N_{G_0}[d] \cup N_{G_0}[e]\big) = \{c, d\}.$$

In either case, $B_t \neq \{d\}$, excluding the final configuration in (5.5).

All possible first-failure configurations are therefore impossible.

Consequently,

$$B_t \neq \varnothing \qquad \forall t \geq 0.$$

These nonempty beliefs give invisible, capture-free robber histories of every finite length against the fixed cop strategy. The histories form a finitely branching rooted tree. By König's infinity lemma, there is an infinite invisible, capture-free play.

Next, take 00001**.** Once again, we label the cycle as $a - b - c - d - e - a$.

Here,

$$E_0 = \{ab, bc, cd, de\}, \qquad E_1 = \{ea\}.$$

The reflection $a \leftrightarrow e$, $b \leftrightarrow d$, fixing c, preserves both snapshots.

Using the same invisible-belief update as above, we prove that $B_t \neq \varnothing$ and

$$\begin{aligned} c_t = b &\implies d \in B_t, \\ (\tau_t, c_t) = (0, c) &\implies B_t = \{a, e\}, \\ (\tau_t, c_t) = (1, a) &\implies c \in B_t, \end{aligned} \tag{5.7}$$

together with their reflected counterparts. These hold initially because $B_0 = A(c_0)$ and $\tau_0 = 0$.

Suppose the assertions hold at round t.

If $\tau_t = 0$, reflection reduces the cop destinations to a, b, c.

If $c_{t+1} = a$, then $c_t \in \{a, b\}$. From $c_t = a$, every position in the nonempty belief $B_t \subseteq \{c, d\}$ can reach c. From $c_t = b$, we get $d \in B_t$, and $dc \in E_0$. Thus $c \in B_{t+1}$.

If $c_{t+1} = b$, then $c_t \in \{a, b, c\}$. From a, every position in $B_t \subseteq \{c, d\}$ can reach d; from b, the possible robber at d can stay; from c, the belief is $\{a, e\}$ and $e \to d$ is legal. Hence $d \in B_{t+1}$.

If $c_{t+1} = c$, then $c_t \in \{b, c, d\}$. From b, the possible robber at d can move to e. From d, $b \in B_t$, so the robber can move from $b \to a$. From c, either position in $B_t = \{a, e\}$ can stay. Since $A(c) = \{a, e\}$, we obtain $B_{t+1} \neq \varnothing$.

These establish the required phase-1 assertions.

If $\tau_t = 1$, the vertices b, c, d are isolated.

At b, the cop must stay, and the possible robber at d can stay, preserving $d \in B_{t+1}$. Reflection handles $c_t = d$.

At c, the cop must stay. Since $\varnothing \neq B_t \subseteq \{a, e\}$ and

$$N_{G_1}[a] = N_{G_1}[e] = \{a, e\},$$

we obtain $B_{t+1} = \{a, e\}$.

At either a or e, the invariant supplies $c \in B_t$. A robber at c can stay and remains invisible from either legal cop destination. Thus $B_{t+1} \neq \varnothing$.

The phase-0 assertions therefore also hold. Hence

$$B_t \neq \varnothing \ \forall t \geq 0.$$

The same König's lemma argument as in Case 1 gives an infinite invisible, capture-free play against every deterministic cop strategy.

So, no deterministic cop strategy guarantees capture at $\ell = 1$ for $q(s) = 2$. □

Theorem 5.11. *Let $s \in \{0, 1\}^5$ define a period-two temporal cycle $C_5(s)$. For every integer $\ell \geq 0$, one cop has a strategy guaranteeing capture if and only if $q(s) = 4$ and $\ell \geq 1$ or $q(s) = 2$ and $\ell \geq 2$.*

Proof. From Lemma 5.7, we know that C_5 is robber-win for $\ell = 0$.

Set $\ell = 1$.

For $q(s) = 4$, we have the schedule $s = 00101$ under reflection, rotation, and phase interchange.

The cop starts at vertex v_2. If the robber chooses v_1 or v_3, then capture occurs immediately. Therefore, the only rational choice for the robber is to choose a vertex from the antipodal set $A_2 = \{v_4, v_5\}$.

The cop moves from $v_2 \to v_1$. After the robber's response in phase 0, $r_1 \in \{v_4, v_5\}$. If $r_1 = v_5$, then capture occurs in the next cop move because τ changes from 0 to 1, making e_5 active.

However, if $r_1 = v_4$, $c_2 = v_5$, $r_2 = v_3$, $c_3 = v_4$, $r_3 = v_2$, $c_4 = v_3$, $r_4 = v_2$, $c_5 = v_2$ and therefore, capture occurs. The robber may choose to remain at the same position, i.e $r_2 = v_4$, or $r_3 = v_3$. However, in each of these scenarios, capture occurs in the next move after the cop moves to same vertex. Therefore, $q(s) = 4$ is cop-win, when $\ell = 1$.

Set $\ell = 2$ and take the C_5 cycle where $q(s) = 2$. The possible schedules then become $s = 00011$ and $s = 00001$.

First, take the schedule $s = 00011$. We label the cycle as: $a - b - c - d - e - a$. This gives us $E_0 = \{ab, bc, cd\}$ and $E_1 = \{de, ea\}$. We take $c_0 = e$. The cop waits the first round through phase 0. After this round, if the robber is a or d, capture occurs immediately. Then to prevent capture the robber must be at $\{b, c\}$. On either position, the robber is forced to wait the round. If the robber is on b, the cop moves $e \to a \to b$ and hence, capture occurs. Similarly, if the robber is at c, to cop moves $e \to d \to c$ and again, capture occurs.

Second, take the schedule $s = 00001$. In this, $E_0 = \{ab, bc, cd, de\}$ and $E_1 = \{ea\}$. We take $c_0 = c$. If the robber starts b or d, capture occurs immediately. Then, r_0 can be a or e. However, by reflection, it suffices to consider $r_0 = a$. The cop moves $c \to b$ in phase 0. The robber is forced to wait. When phase changes to 1, the cop must wait. If the robber waits at a, then the cop can capture in the next move $b \to a$. Therefore, to survive, the robber must move $a \to e$. Then, when the phase becomes 0, the cop moves $b \to a$ and the robber moves $e \to d$, because if the robber stays at e, it permits capture in the next move. Then the phase changes to 1. The cop can move $a \to e$ while the robber must wait. This allows the cop to capture the robber in the next move $e \to d$.

Hence, $q(s) = 2$ is cop-win when $\ell = 2$. ■

5.8. Cycle C_6 and C_7

Lemma 5.12. *For the period-two temporal cycles $C_6(001001)$ and $C_6(010101)$, the robber can evade capture indefinitely for every visibility radius $\ell \geq 0$. The same holds for schedules equivalent to these under rotation, reflection, or phase interchange.*

Proof. Label the vertices $v_1, v_2, \ldots, v_6$ with indices interpreted modulo 6.

First, let $s = 001001$**.**

We label the phase assigned to each edge i as, s_i. Since $s_i = s_{i+3}$, rotation by 3 vertices preserves both active snapshots.

If the cop chooses $c_t = v_i$ at time t, the robber chooses their position as $r_t = v_{i+3}$. At $c_{t+1} = v_j$, then the robber chooses $r_{t+1} = v_{j+3}$. This response is legal because rotation preserves the active edges. If the cop stays, then the robber also stays.

Since the cop moves at most one edge, this means $d(c_{t+1}, r_t) \geq 2$. Thus, the cop does not capture the robber. After the robber's response, $d(c_{t+1}, r_{t+1}) = 3$. Therefore, this strategy avoids capture indefinitely.

Second, let $s = 010101$**.**

Here, $s_i = s_{i+2}$, so rotation by two vertices preserves the active snapshots. The robber starts two vertices ahead of the cop and maintains. So, $c_t = v_i \implies r_t = v_{i+2}$ and after the cop moves $c_{t+1} = v_j$, the robber chooses $r_{t+1} = v_{j+2}$.

After the cop's move, $d(c_{t+1}, r_t) \geq 1$, so capture does not occur. After the robber moves, the new distance becomes $d(c_{t+1}, r_{t+1}) = 2$. Therefore, there exists indefinite evasion.

Neither strategy depends on the visibility radius. □

Lemma 5.12 is represented by Figure 3.

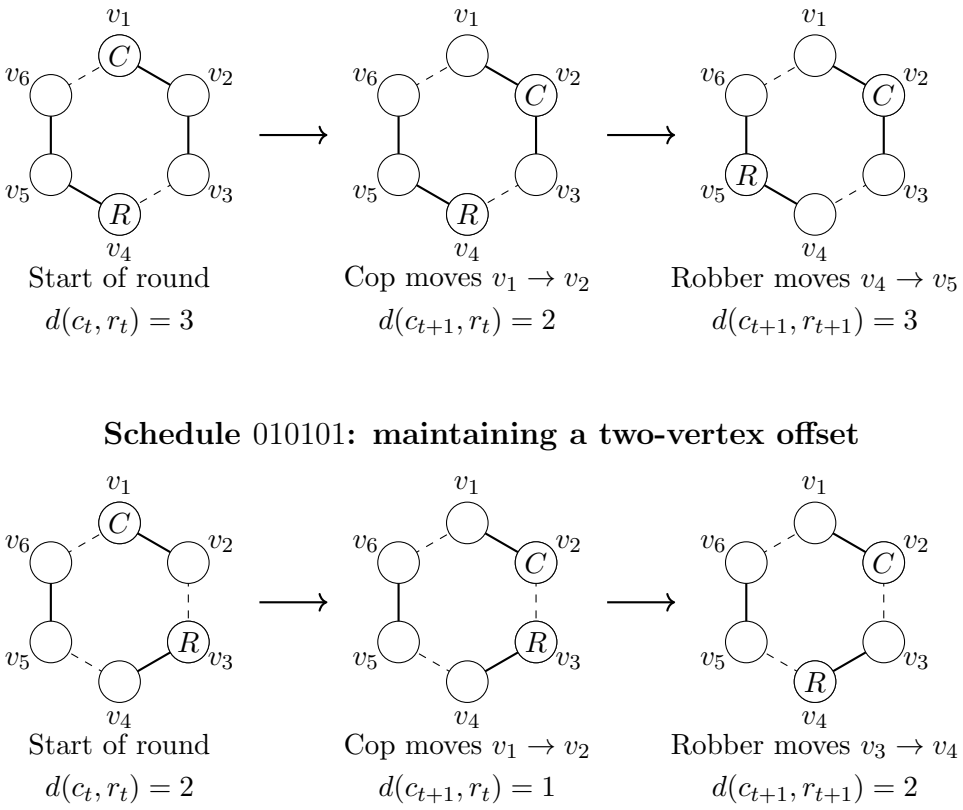

Figure 3. One round of each evasion strategy on C_6. Solid edges are active in phase 0 and dashed edges are inactive in phase 0.

Theorem 5.13. *Let $s \in \{0,1\}^n$, represent a period-two cycle $C_n(s)$, where $n \in \{6,7\}$. For every integer $\ell \geq 0$: (a) if $n = 6$, a cop has a deterministic strategy that guarantees capture if and only if $\ell \geq 2$ and either $s = 000101$ or $s = 001011$, under rotation, reflection, and phase interchange; and (b) if $n = 7$, a cop has a deterministic strategy that guarantees capture if and only if $\ell \geq 3$ and $q(s) = 6$.*

Proof. For a schedule s, an information state is represented as $S = (c, B, \tau)$, where c is the cop's current position, B is the belief state, and τ is the current phase. For a legal cop destination c', let $\mathrm{Succ}(S, c')$ contain every non-capture successor state after a legal robber response.

Let W_k denote the set of information states from which the cop can guarantee capture within k additional rounds. So $W_0 = \varnothing$. Then we define,

$$W_{k+1} = W_k \cup \{S : \exists c' \in N_{G_\tau}[c] | \forall S' \in \mathrm{Succ}(S, c'), S' \in W_k\}.$$

Since there are only a finite number of information sets, eventually $W_{k+1} = W_k$. We call this final set W^*. Then, we enumerate all schedules in $n \in \{6,7\}$. This gives us the winning

schedules for C_6 as 000101 and 001011, and their winning radius is $\ell \geq 2$. For C_7, the exact schedules that are winning are those with $q(s) = 6$ and their winning radius is $\ell \geq 3$.

The source code and detailed results are provided in the Appendix A. ■

The visibility radius has to be $\ell \geq 3$ for a C_7 cycle. This means the cop must have full visibility because the diameter of C_7 is 3. C_6 with $\ell \geq 2$ is different because it also has a diameter of 3. This means it genuinely has limited-visibility win.

5.9. Cycle C_8 and beyond

Theorem 5.14. *Let $n \geq 8$. Let $s \in \{0,1\}^n$ define a period-two temporal cycle $C_n(s)$. For every integer $\ell \geq 0$, no deterministic cop strategy guarantees capture.*

Proof. We use [ES19, Theorem 13] to prove this. Their Theorem 13 states that:

If $n \geq 2l\,\mathrm{LCM}(L)$, then the edge-periodic cycle is robber win, where

$$l = \begin{cases} 1, \mathrm{LCM}(L) \geq 2 \max L, \\ 2, \text{otherwise.} \end{cases}$$

For our model, $L = \{2\}$, $\mathrm{LCM}(L) = 2$, and $\max L = 2$. This means $l = 2$, as well. Hence, $n \geq 2(2(2)) = 8$. The cited result establishes evasion even with full visibility. This means the theorem applies to every C_n, with $n \geq 8$ and $\forall \ell \in \mathbb{Z}_{\geq 0}$. ■

6. Conclusion

This paper studies Cops and Robbers under two simultaneous restrictions: periodically changing edge availability and limited visibility. For temporal paths, the cop has a *sweeping* strategy to always win for any large n and any ℓ. As a result, every temporal path is a cop-win provided that each edge appears at least once every period.

Therefore, we introduce another complication. We see how this differs in temporal cycles. In cycles the robber has the ability to move to the initial vertex even after moving to the last vertex. This prevents the shrinking argument used in proving the cop-win nature of temporal paths. For period-2 temporal cycles with an arbitrary visibility radius ℓ, the results show that C_3 is cop-win exactly when either $q(s) = 2$ and $\ell \geq 0$, or $q(s) = 0$ and $\ell \geq 1$. We also prove that for every $n \geq 4$, a cycle C_n is always robber-win when the visibility radius $\ell = 0$. C_4 is cop-win exactly when $q(s) = 2$ and $\ell \geq 1$. C_5 is cop-win exactly when either $q(s) = 4$ and $\ell \geq 1$ or $q(s) = 2$ and $\ell \geq 2$. For C_6 and C_7, through exhaustion, we show that C_6 is cop-win when the visibility radius is $\ell \geq 2$ and the schedule is either 000101 or 001011, up to rotation, reflection, and phase interchange. For C_7, we show that for a visibility radius $\ell \geq 3$ and $q(s) = 6$. This is important because $q(s)$ stops working at C_6.

Using the result from Erlebach and Spooner [ES19], we show that every cycle is robber-win for $n \geq 8$.

These results show that temporal structure alone does not determine the outcome of pursuit. The interaction between information availability and graph topology is also important. Limited visibility serves as an advantage to the robber as the graph grows in size, while smaller cycles remain sensitive to the arrangement of the temporal phases.

Extensions to this paper can include larger periods, multiple cops, and other graph families.

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A. Exhaustive Proof for C_6 and C_7

This appendix outlines the exact solver used to prove cycles C_6 and C_7 in section 5.8.

A.1. Game conventions and information states

Label the vertices $v_1, \dots, v_n$, and let

$$e_i = \{v_i, v_{i+1}\}, \qquad v_{n+1} = v_1.$$

Visibility is measured in the underlying cycle:

$$d_{C_n}(v_i, v_j) = \min\{|i-j|,\, n-|i-j|\}.$$

For a fixed radius ℓ, write

$$V_\ell(c) = \{r \in V : d_{C_n}(c, r) \le \ell\}, \qquad I_\ell(c) = V \setminus V_\ell(c).$$

A non-capture information state is

$$S = (c, B, \tau),$$

where $c \in V$ is the cop's position, $\tau \in \{0, 1\}$ is the phase, and

$$\varnothing \neq B \subseteq V \setminus \{c\}$$

is the set of possible robber positions consistent with the observation history.

A visible observation gives a singleton belief. An invisible observation gives a nonempty subset of $I_\ell(c)$. Accordingly, the solver enumerates the finite state space

$$\mathcal{S}_\ell = \left\{ (c, B, \tau) : \begin{array}{l} c \in V,\ \tau \in \{0,1\}, \\ B = \{r\} \text{ for some } r \neq c, \text{ or} \\ \varnothing \neq B \subseteq I_\ell(c) \end{array} \right\}.$$

A.2. Successor-state construction

Fix a state $S = (c, B, \tau)$ and a legal cop destination

$$u \in N_{G_\tau}[c].$$

After the cop's move and the robber's response, the set of possible surviving robber positions is

$$U = \left(\bigcup_{r \in B \setminus \{u\}} N_{G_\tau}[r] \right) \setminus \{u\}.$$

Every visible position $r \in U \cap V_\ell(u)$ produces the successor state

$$(u, \{r\}, 1-\tau).$$

If $U \cap I_\ell(u) \neq \varnothing$, the invisible observation produces the single successor

$$(u, U \cap I_\ell(u), 1-\tau).$$

Thus,

$$\begin{aligned}\mathrm{Succ}(S,u) = \ & \{(u,\{r\},1-\tau) : r \in U \cap V_\ell(u)\} \\ & \cup \begin{cases} \{(u, U\cap I_\ell(u), 1-\tau)\}, & U \cap I_\ell(u) \neq \varnothing, \\ \varnothing, & U \cap I_\ell(u) = \varnothing. \end{cases}\end{aligned}$$

If $\mathrm{Succ}(S,u) = \varnothing$, no robber play survives the round after that cop move.

A.3. Correctness of the fixed-point method

Define

$$W_0 = \varnothing$$

and

$$W_{k+1} = W_k \cup \{S \in \boldsymbol{S}_\ell : \exists u \in N_{G_\tau}[c]\ \forall S' \in \mathrm{Succ}(S,u),\ S' \in W_k\}.$$

We prove that W_k consists exactly of the non-capture information states from which the cop can guarantee capture within at most k additional rounds.

For $k = 0$, no non-capture state permits capture in zero additional rounds, so $W_0 = \varnothing$.

Suppose the statement holds for k. If a state is added to W_{k+1}, the cop can choose a move whose every non-capture successor lies in W_k. After observing the resulting state, the cop can therefore guarantee capture within k further rounds. If there are no non-capture successors, capture is guaranteed during the current round.

Conversely, suppose a strategy guarantees capture within $k+1$ rounds. Its first move must leave every possible non-capture successor in a state from which capture can be guaranteed within k further rounds. By the induction hypothesis, every such successor belongs to W_k. Therefore, the original state belongs to W_{k+1}.

This completes the induction.

Since

$$W_k \subseteq W_{k+1} \subseteq \boldsymbol{S}_\ell$$

and $\boldsymbol{S}_\ell$ is finite, the sequence stabilizes at a fixed point W^*.

Every state in W^* admits a strategy with a finite capture bound. Conversely, suppose a strategy guarantees eventual capture but has no finite uniform capture bound. Its capture-free play histories then form a finitely branching rooted tree with paths of arbitrarily large finite length. By König's infinity lemma, this tree has an infinite path, contradicting guaranteed capture.

Hence,

$$S \in W^* \quad \Longleftrightarrow \quad \text{the cop can guarantee capture from } S.$$

A.4. Initial-position test

For an initial cop vertex c, let $\boldsymbol{\mathcal{I}}_\ell(c)$ be the set of states arising from all possible initial observations:

$$\begin{aligned} \boldsymbol{\mathcal{I}}_\ell(c) = \; & \{(c, \{r\}, 0) : r \in V_\ell(c) \setminus \{c\}\} \\ & \cup \begin{cases} \{(c, I_\ell(c), 0)\}, & I_\ell(c) \neq \varnothing, \\ \varnothing, & I_\ell(c) = \varnothing. \end{cases} \end{aligned}$$

The schedule is cop-win precisely when

$$\exists c \in V \quad \text{such that} \quad \boldsymbol{\mathcal{I}}_\ell(c) \subseteq W^*.$$

The existential choice of c represents the cop choosing their starting vertex. Requiring every initial observation state to lie in W^* accounts for every possible initial robber position.

A.5. Enumeration and implementation

The solver enumerates every schedule in $\{0,1\}^n$ for $n \in \{6,7\}$. For each schedule, it checks

$$\ell = 0, 1, 2, 3.$$

Since both cycles have diameter 3, larger visibility radii produce the same observations as $\ell = 3$.

The computation therefore checks

$$4(2^6 + 2^7) = 768$$

schedule-radius combinations. No schedule is omitted through symmetry reduction.

Belief sets are stored as integer bitmasks. The implementation uses vertex indices $0, \ldots, n-1$, corresponding to $v_1, \ldots, v_n$ in the mathematical notation.

The fixed point is computed using a queue. For every state and legal cop move, the implementation records the number of distinct non-capture successor states not yet marked winning. Moves with no non-capture successors initialize the queue. Whenever a state becomes winning, the counters of its predecessor actions are reduced. An action becomes winning when its counter reaches zero, at which point its owner state is marked winning.

This implements the same closure rule as the sequence $(W_k)_{k \geq 0}$, without explicitly storing the layers.

The program also checks visibility monotonicity:

$$\text{cop-win at } \ell \implies \text{cop-win at } \ell + 1.$$

This is an implementation consistency check; it does not replace the correctness argument above.

A.6. Detailed results

The following table gives the number of schedules admitting a deterministic cop strategy guaranteeing capture.

Cycle	ℓ	Cop-win	No guaranteed capture
C_6	0	0	64
C_6	1	0	64
C_6	2	24	40
C_6	3	24	40
C_7	0	0	128
C_7	1	0	128
C_7	2	0	128
C_7	3	14	114

Table 1. Classification by cycle length and visibility radius.

Define $\ell_*(s)$ as the smallest winning visibility radius, with $\ell_*(s) = \infty$ when no radius permits guaranteed capture. The following tables group the schedule-level results under rotation, reflection, and phase interchange. Class size counts distinct binary schedules.

Representative	$q(s)$	Class size	$\ell_*(s)$
000000	0	2	∞
000001	2	12	∞
000011	2	12	∞
000111	2	6	∞
000101	4	12	2
001001	4	6	∞
001011	4	12	2
010101	6	2	∞

Table 2. Schedule classes for C_6.

Representative	$q(s)$	Class size	$\ell_*(s)$
0000000	0	2	∞
0000001	2	14	∞
0000011	2	14	∞
0000111	2	14	∞
0000101	4	14	∞
0001001	4	14	∞
0001011	4	28	∞
0010011	4	14	∞
0010101	6	14	3

Table 3. Schedule classes for C_7.

The CSV records one row per schedule-radius combination, with columns

$$n, \quad s, \quad q(s), \quad \ell, \quad \texttt{cop_win}, \quad \ell_*(s).$$

The value `cop_win`=1 indicates guaranteed capture, while `cop_win`=0 indicates that no deterministic cop strategy guarantees capture. The value `infinity` denotes $\ell_*(s) = \infty$.

A.7. Source code

Listing 1. Exact belief-state solver for C_6 and C_7

```
1
2  from itertools import product
3  from collections import defaultdict, deque
4  import csv
5
6  N_VALUES = (6, 7)
7
8
9  def vertices(mask):
10     while mask:
11         bit = mask & -mask
12         yield bit.bit_length() - 1
13         mask ^= bit
14
15
16 def distance(n, u, v):
17     d = abs(u - v)
18     return min(d, n - d)
19
20
21 def q_value(schedule):
22     n = len(schedule)
23     return sum(
24         schedule[i] != schedule[(i + 1) % n]
25         for i in range(n)
26     )
27
28
29 def solve_schedule(schedule, ell):
30     n = len(schedule)
31
32     moves = [[{v} for v in range(n)] for _ in range(2)]
33     for u, phase in enumerate(schedule):
34         v = (u + 1) % n
35         moves[phase][u].add(v)
36         moves[phase][v].add(u)
37
38     hidden = [
39         sum(
40             1 << r
41             for r in range(n)
42             if distance(n, c, r) > ell
43         )
44         for c in range(n)
45     ]
```

```
46
47      states = []
48      for c in range(n):
49          beliefs = {1 << r for r in range(n) if r != c}
50
51          subset = hidden[c]
52          while subset:
53              beliefs.add(subset)
54              subset = (subset - 1) & hidden[c]
55
56          for phase in (0, 1):
57              states.extend(
58                  (c, belief, phase)
59                  for belief in beliefs
60              )
61
62      def successors(state, cop_next):
63          _, belief, phase = state
64          possible = 0
65
66          for r in vertices(belief):
67              if r == cop_next:
68                  continue
69
70              for robber_next in moves[phase][r]:
71                  if robber_next != cop_next:
72                      possible |= 1 << robber_next
73
74          result = set()
75          invisible = possible & hidden[cop_next]
76
77          if invisible:
78              result.add(
79                  (cop_next, invisible, 1 - phase)
80              )
81
82          visible = possible & ~hidden[cop_next]
83          for r in vertices(visible):
84              result.add(
85                  (cop_next, 1 << r, 1 - phase)
86              )
87
88          return result
89
90      owners = []
91      remaining = []
92      predecessors = defaultdict(list)
93      winning = set()
94      queue = deque()
95
96      for state in states:
97          c, _, phase = state
98
99          for cop_next in moves[phase][c]:
```

```
            outcomes = successors(state, cop_next)
            action_id = len(owners)
            owners.append(state)
            remaining.append(len(outcomes))

            for outcome in outcomes:
                predecessors[outcome].append(action_id)

            if not outcomes and state not in winning:
                winning.add(state)
                queue.append(state)

    while queue:
        newly_winning = queue.popleft()

        for action_id in predecessors[newly_winning]:
            remaining[action_id] -= 1

            if remaining[action_id] == 0:
                state = owners[action_id]
                if state not in winning:
                    winning.add(state)
                    queue.append(state)

    for c in range(n):
        initial_states = []

        if hidden[c]:
            initial_states.append(
                (c, hidden[c], 0)
            )

        for r in range(n):
            if r != c and distance(n, c, r) <= ell:
                initial_states.append(
                    (c, 1 << r, 0)
                )

        if all(
            state in winning
            for state in initial_states
        ):
            return True

    return False


def main():
    with open(
        "cycle_visibility_results.csv",
        "w",
        newline="",
        encoding="utf-8"
    ) as file:
```

```
        writer = csv.writer(file)
        writer.writerow([
            "n", "schedule", "q",
            "ell", "cop_win", "ell_star"
        ])

        for n in N_VALUES:
            diameter = n // 2

            for schedule in product((0, 1), repeat=n):
                results = [
                    solve_schedule(schedule, ell)
                    for ell in range(diameter + 1)
                ]

                assert all(
                    not results[ell] or results[ell + 1]
                    for ell in range(diameter)
                ), (
                    f"Monotonicity failed: "
                    f"n={n}, s={schedule}"
                )

                ell_star = next(
                    (
                        ell
                        for ell, win in enumerate(results)
                        if win
                    ),
                    None
                )

                threshold = (
                    "infinity"
                    if ell_star is None
                    else ell_star
                )

                word = "".join(map(str, schedule))
                q = q_value(schedule)

                for ell, cop_win in enumerate(results):
                    writer.writerow([
                        n, word, q, ell,
                        int(cop_win), threshold
                    ])


if __name__ == "__main__":
    main()
```

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